

## Review for Unit 2: Exponentials & Logarithms Exam

**Logistics of the Exam** – The exam will be 20 questions. There will be two sections: Section A will include 10 multiple choice questions in which you will be allowed to use a calculator and worth 40 points. Section B will include 10 free response questions in which you will not be allowed to use a calculator and worth 60 points. The multiple-choice questions will have 4 to 5 answer choices. The free response questions will be a mix of error analysis, modelling, and short answer. You will have 80 minutes to complete the entire exam and you may not have more time. Once you complete Section A, you will not be allowed to go back to it.

**Resources and Tools Allowed on the Exam** – pencil, highlighter, Unit 2 formulas (given in the box below) and a graphic display calculator (for Section A).

<p style="text-align: center;"><b>Definition of a Logarithm</b></p> $b^k = a \text{ iff } \log_b a = k$ <p style="text-align: center;"><b>Compound Interest</b></p> $A = P \left( 1 + \frac{r}{n} \right)^{nt}$	<p style="text-align: center;"><b>Properties of Logarithms</b></p> $\log_b(u \cdot v) = \log_b u + \log_b v$ $\log_b \left( \frac{u}{v} \right) = \log_b u - \log_b v$ $\log_b u^k = k \log_b u$
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**Content of the Exam** – Students will be expected to know how to:

- Compose two functions. [MC]
- Evaluate the composition of two functions for a given input. [MC]
- Given the table of a function, complete the table for the inverse of that function. [FR]
- Find the equation of the inverse of a function. [FR]
- Give an exponential model/function for a table of values. [FR]
- Evaluate logarithms. [MC & FR]
- Switch from exponential form to logarithmic form and vice versa. [FR]
- Describe the characteristics of an exponential function including but not limited to domain, range, intercepts, asymptote and shape. [MC]
- Describe the transformations from one exponential function to another exponential function. [MC]
- Solve a one-to-one exponential equation. [FR]
- Solve a one-to-one logarithmic equation. [FR]
- Solve an exponential equation using inverse properties. [FR]
- Solve a logarithmic equation using inverse properties. [FR]
- Expand a single logarithm using the properties of logarithms. [FR]
- Condense an expression with logarithms into a single logarithm using properties of logarithms. [FR].
- Find the account balance of an investment compounded n times per year or continuously. [MC]
- Given the half-life of a substance, calculate how much is remaining after a certain amount of time. [MC].

**From Unit 1:**

- Given a data graph, state which measure of center and spread is best based on its shape and why. [FR].
- Use z-scores to compare two data values in separate situations. [MC].
- Find the probability of getting a data value greater than a number from a normal distribution. [MC]
- Given a statistical study, state which type of study it is and the population of interest. [FR].

These practice problems are to help you with the skills on the exam. It is not inclusive of what the questions will be actual exam.

Rewrite the logarithmic equation in exponential form.

1)  $\log_u v = \frac{5}{12}$

2)  $\log_{11} 55 = p$

Rewrite the exponential equation in logarithmic form.

3)  $b^a = 116$

4)  $n^{-4} = \frac{3}{4}$

Give the characteristics of the function graphed to the right.

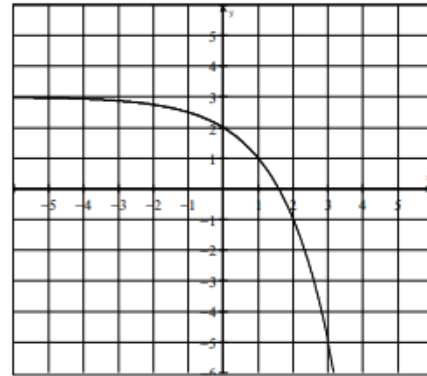
5)

Domain: \_\_\_\_\_ Range: \_\_\_\_\_

x-int: \_\_\_\_\_ y-int: \_\_\_\_\_

equation of asymptote: \_\_\_\_\_ H V

growth or decay? \_\_\_\_\_



Use a graphic display calculator to evaluate the logarithmic expressions below. Give your answer correct to 3 significant figures.

6)  $\log_3 47 + \ln 7.8$

7)  $\log 52 - \log_4 1080$

Use properties of logs to expand the logarithm completely. Simplify, if necessary. There should be no factors, powers, or fractions in any logarithm in your final answer.

8)  $\ln(x \cdot e^4)$

9)  $\log\left(\frac{y^2}{10}\right)$

Use the properties of logs to condense the expression into a single logarithm. Be sure to simplify, if necessary.

10)  $\log(8a) - \log(2b)$

11)  $\log_2 8 - 2 \log_2 2 + \log_2 3$

Solve the exponential equations below.

12) Solve:  $2^{3x+3} = 8^{2x}$

13) Solve.  $27^{x+2} = 9^{2x-1}$

14) Solve:  $5^{2x} = 12$ . Give the answer exactly and rounded to three decimal places.

15) Solve:  $e^{4x-5} - 7 = 11243$ . Give the answer exactly and rounded to three decimal places.

Solve the logarithmic equations. Be sure to check for extraneous solutions.

16) Solve.  $\log_{15}(2m + 3) = \log_{15}(m + 12)$

18) Solve.  $\log_2 x + 5 = 7$

20) Suppose Jackson would like to invest \$40,000 in a CDA that offers 9.2% interest per annum compounded monthly. a) Write a model for the future value of his investment. b) How much money will be in the CDA after 5 years?

22) A 90-mg sample of radioactive isotope has a half-life of 5 years. A) Write a function that relates the mass of the sample,  $m$ , to the elapsed time,  $t$ . b) How much substance is left after 3 years?

24) The graphs of  $y = 3^x$  and  $y = \log_3 x$  are: a) reflected across the line  $y=0$ , b) reflected across the line  $x = 0$ , c) reflected across the line  $y = x$ , or d) identical

26) What transformation maps  $f(x) = 3^x$  to  $g(x) = 3^{-x}$ ?

28) Test scores are normally distributed with a mean of 79 and a standard deviation of 8. What is the probability that a student passed the test (passing score is 70)?

17) Solve.  $\log_{11}(36 - x) = \log_{11}(42 - 3x)$

19) Solve.  $\log_2 x + \log_2 3 = 3$

21) Michelle is a sophomore in college opening up her first credit card account. She spends \$500 on the card. a) If the card has an APR of 31.99% compounded monthly, write a model for the future value of the charge (with interest). b) How long will it take the charge to double assuming no payments are made? Round to the nearest tenth.

23) There is a sale at a store for 10% off your purchase and you have a coupon for \$50 off your purchase. Write functions  $f(x)$  and  $g(x)$  to represent the 10% of the price  $x$  of an item and the \$50 coupon respectively. A) Find  $f(g(x))$ . If you are purchasing a \$1000 stereo, how much will you pay for it?

25) Find the inverse of  $f(x) = 3^x + 4$

27) If a student scored a 28 on a Chemistry test where the class had a mean of 30 and a standard deviation of 4 and then scored a 52 on a Math test where the class had a mean of 60 and a standard deviation of 10. On which test, did he score better in relation to his peers?

Classify the situations as sample survey, observational study, or experimental study:

29) A drug is given to 15 patients and a placebo to another group to determine its effect on an illness.

30) A researcher conducts a study at the mall counting the number of men that wash their hands after using the restroom.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. Judy works for a doctor. She placed a sample of bacteria in a culture dish and recorded the number of bacteria present each 30 minutes beginning at 12:00 P.M. The table shows Judy's data.

**Bacterial Growth**

Time	Number of Bacteria Present
12:00 P.M.	150
12:30 P.M.	600
1:00 P.M.	2400

If the pattern of bacterial growth remains constant, how many bacteria should be present in the culture dish at 2:00 P.M.?

2. Which is the first *incorrect* step in simplifying  $\log_4 \frac{4}{64}$ ?

Step 1:  $\log_4 \frac{4}{64} = \log_4 4 - \log_4 64$   
 Step 2:  $= 1 - 16$   
 Step 3:  $= -15$

- (A) Step 1 (B) Step 2  
 (C) Step 3 (D) Each step is correct.

3. What is the equation of the function represented by this table of values?

$x$	-2	-1	0	1	2
$y$	$\frac{3}{25}$	$\frac{3}{5}$	3	15	75

- (A)  $y = 5x + 3$  (B)  $y = 12x + 3$   
 (C)  $y = 3 \cdot 5^x$  (D)  $y = 5 \cdot 3^x$

4. If  $f(x) = x^2 + 3$  and  $g(x) = x - 2$ , find  $(f \circ g)(2)$ .

5. Which of the following is the logarithmic form of the equation  $y = 20^{\frac{-3}{2}}$ ?

- (A)  $\log_{20} y = \frac{-3}{2}$  (B)  $\log_{\frac{3}{2}} 20 = y$   
 (C)  $-\log_{\frac{3}{2}} y = 20$  (D)  $\log_{20}(\frac{-3}{2}) = y$

6. Which equation is equivalent to  $3 \log x + \log 2 = \log 3x - \log 2$ ?

- (A)  $\log x^3 + 2 = \log(3x - 2)$  (B)  $\log(3x + 2) = \log(3x - 2)$   
 (C)  $\log 6x = \log\left(\frac{3x}{2}\right)$  (D)  $\log(2x^3) = \log\left(\frac{3x}{2}\right)$

7. Which statement about the graph of the equation  $y = e^x$  is *not* true?

- (A) It is asymptotic to the  $x$ -axis.  
 (B) The domain is the set of all real numbers.  
 (C) It lies in Quadrants I and II.  
 (D) It passes through the point  $(e, 1)$ .

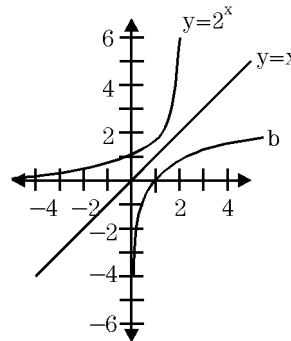
8. The surface area of a balloon can be represented by the function  $S(r) = 4\pi r^2$ , where  $r$  is the radius of the balloon. If  $r$  increases with time,  $t$ , and is represented by the function  $r(t) = \frac{1}{4}t^2$ , what is the surface area of the balloon expressed as a function of time?

- (A)  $S(t) = 4\pi t^2$  (B)  $S(t) = \pi t^2$   
 (C)  $S(t) = \frac{\pi^4}{4}$  (D)  $S(t) = \frac{\pi^2 t^2}{16}$

9. If the graph of the equation  $y = 3^x$  is reflected in the  $x$ -axis, the equation of the reflection is

- (A)  $y = 3^{-x}$  (B)  $y = -(3^x)$   
 (C)  $y = \log_x 3$  (D)  $y = x^3$

10. In the diagram shown, figure  $b$  is the reflection of  $y = 2^x$  in the line  $y = x$ .



Which is an expression for the equation of figure  $b$ ?

- (A)  $y = (-2)^x$  (B)  $y = 2^{-x}$   
 (C)  $y = \log_2 x$  (D)  $y = \log_x 2$

11. Which transformation best describes the relationship between the functions  $f(x) = 2^x$  and  $g(x) = (\frac{1}{2})^x$ ?

- (A) reflection in the line  $y = x$   
 (B) reflection in the origin  
 (C) reflection in the  $x$ -axis  
 (D) reflection in the  $y$ -axis