

# 4.9 The Zeros of Polynomials

Activator: Zero Zappers Diagnostic

1. How many zeros does the polynomial function have?

**2**

2. What are the zeros of this polynomial function?

**$x = 6, 8$**

3. Write a possible equation for this graph.

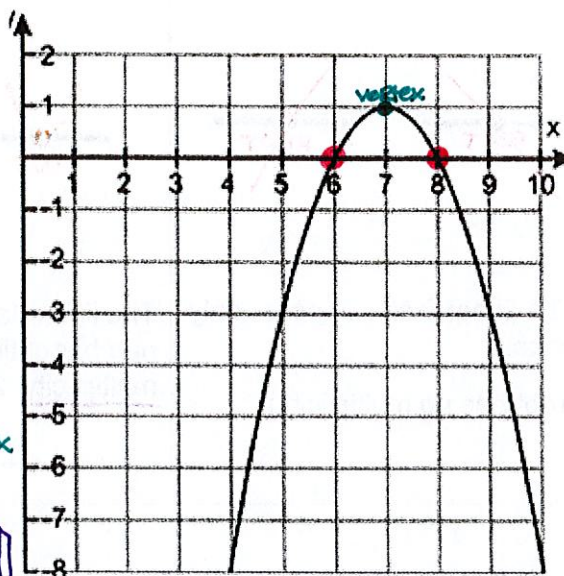
**factored form**  $y = -(x-6)(x-8)$  or **vertex form**  $y = -(x-7)^2 + 1$

4. Write the possible question in standard form. *Since (7, 1) is the vertex*

$y = -(x-6)(x-8)$

$y = -(x^2 - 8x - 6x + 48)$

$y = -(x^2 - 14x + 48) \rightarrow y = -x^2 + 14x - 48$



Engage: Zero Zappers Desmos Activity

From the FUNDAMENTAL THEOREM OF ALGEBRA:

"An 'nth' degree polynomial function has exactly 'n' zeros in the set of complex numbers, counting repeated zeros." Complex zeros can be Real zeros or Imaginary zeros. Imaginary zeros always come in pairs. Zeros of a function occur when the function is equal to zero. Therefore, if the zero is Real it will cross the x-axis at that number.

Write a summary about different functions and the possible number of real zeros below. **n = degree**

| DEGREE | NAME      | # of POSSIBLE REAL ZEROS |
|--------|-----------|--------------------------|
| 1      | LINEAR    | 1 real zero              |
| 2      | QUADRATIC | 0 or 2 Real zeros        |
| 3      | CUBIC     | 1 or 3 real zeros        |
| 4      | QUARTIC   | 0 or 2 or 4 real zeros   |
| 5      | QUINTIC   | 1 or 3 or 5 real zeros   |

The possible # of real zeros for an even-degree polynomial function is 0, 2, 4, up to n.

The possible # of real zeros for an odd degree polynomial function is 1, 3, 5 ... up to n.

Mini-Lesson: Fundamental Theorem of Algebra (FTA)

**First Check:** How many roots should the polynomial have according to FTA? **1, 3, 5 ... up to n.**

|                          |          |                    |          |                      |          |
|--------------------------|----------|--------------------|----------|----------------------|----------|
| a) $x^3 + 5x^4 - 2x + 1$ | <b>4</b> | b) $4x^2 + 5x + 7$ | <b>2</b> | c) $x^6 - 5x^3 - 14$ | <b>6</b> |
|--------------------------|----------|--------------------|----------|----------------------|----------|

**DEGREE = the Biggest exponent in the polynomial**

A **root** by definition is a number when substituted for the variable yields an output of zero. Roots are sometimes called complex solutions. When the roots are repeated, we call this **multiplicity**.



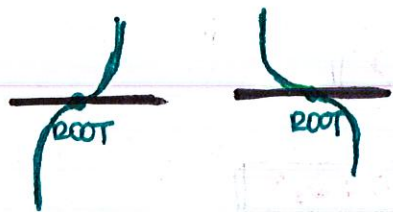
**Remember:** If the root, a, is a real number then the following statements are equivalent:

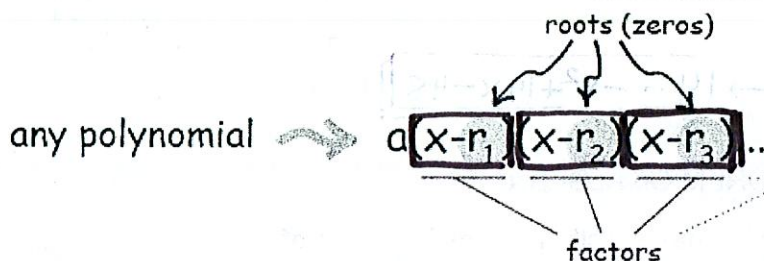
a is a **zero** of the function  $f(x)$  | a is a **solution** to the equation  $f(x) = 0$  |  $(x - a)$  is a **factor** of the polynomial  $f(x)$  |  $[(a, 0)$  is an **x-intercept** on the graph of  $f$ ]\*

\*For complex roots, the last statement will not be valid since we cannot graph complex numbers on a real number line.

## ON THE GRAPH!

Let's talk about the three different things that can happen at a **real ROOT**. The graph could:

|   |  |  |
|---|--|--|
| <p>Go straight through like a <b>LINE</b>,...</p>  <p>The linear factor appears only once.<br/>(root has no multiplicity)</p> | <p>touch the x-axis and <b>BOUNCE</b>,...</p>  <p>The linear factor appears an even number of times (root has even <u>multiplicity</u>: 2, 4, 6...)</p> | <p>or flatten out near the axis like a <b>WIGGLE</b>.</p>  <p>The linear factor appears an odd number of times greater than or equal to 3 (root has odd multiplicity: 3, 5, 7...)</p> |
|---|--|--|



- repeated roots will have linear factors with exponents on them.
- if the polynomial has complex roots, two linear factors will pair up to an irreducible quadratic with real coefficients.
- The degree can be determined from factored form by adding the exponents of every linear factor together. Please note  $(x - 0)$  just becomes  $x$ .

Use the factored forms below to give the degree of the polynomial function. Then use the zero product property to find all of the roots.

|  |   |   |
|--|---|---|
| <p>d) <math>p(x) = 2x(x+4)^2(x-7)</math><br/> <span style="margin-left: 40px;">① ②③ ④</span></p> <p><b>degree: 4</b></p> <p><math>0 = 2x(x+4)^2(x-7)</math></p> <p><math>2x=0 \quad x+4=0 \quad x-7=0</math></p> <p><math>x=0 \quad x=-4 \quad x=7</math></p> <p><b>Roots: 0, 7, -4 with mult. 2</b></p> | <p>e) <math>f(x) = (x+5)(x-3)(x+2)</math><br/> <span style="margin-left: 40px;">① ② ③</span></p> <p><b>degree: 3</b></p> <p><math>0 = (x+5)(x-3)(x+2)</math></p> <p><math>x+5=0 \quad x-3=0 \quad x+2=0</math></p> <p><math>x=-5 \quad x=3 \quad x=-2</math></p> <p><b>roots: -5, 3, -2</b></p> | <p>f) <math>g(x) = 3(x-6)^3(x+1)</math><br/> <span style="margin-left: 40px;">①②③ ④</span></p> <p><b>degree: 4</b></p> <p><math>0 = 3(x-6)^3(x+1)</math></p> <p><math>x-6=0 \quad x+1=0</math></p> <p><math>x=6 \quad x=-1</math></p> <p><b>roots: -1, 6 with mult. 3</b></p> |
|--|---|---|

**To find degree:** count how many linear factors are in factored form  
**To find the roots:** Set equation equal to zero & each linear equal to zero & solve for  $x$  (don't forget to state any multiplicities)