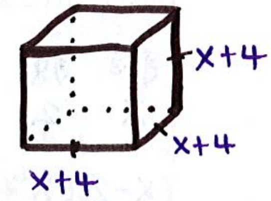
Name: ______________________ Date: ____________ Period: ________

4.8 **Factoring Polynomials (3+ Degree)**

Engage: Cubic Identities

1. What is the volume of a cube with side length equal to 4? $4^3 = \boxed{64}$

2. What is the volume of a cube with side length equal to x? $\boxed{x^3}$



3. Now we will determine the volume of a cube with side length equal to $x + 4$.

   a. First, use the rule for squaring a sum to find the area of the base of the cube.

   $(x+4)(x+4) = (x+4)^2 = x^2 + 2\cdot x\cdot 4 + 4^2 = \boxed{x^2+8x+16}$

   factored form of PST

   b. Now use the distributive property to multiply the area of the base by the height $(x + 4)$ and simplify your answer.

   $(x+4)(x^2+8x+16) = x^3 + 8x^2 + 16x + 4x^2 + 32x + 64$

   $= \boxed{x^3+12x^2+48x+64}$

4. What is the volume of a cube with side length equal to $x + y$? Use the same steps as in Step 3 to determine this.

	x^2	$+2xy$	$+y^2$
x	x^3	$2x^2y$	xy^2
y	x^2y	$2xy^2$	y^3

$= \boxed{x^3 + 3x^2y + 3xy^2 + y^3}$

5. So the identify for a binomial cube is: $(x + y)^3 =$ $x^3 + 3x^2y + 3xy^2 + y^3$

6. Determine the following identity: $(x - y)^3 =$ $x^3 - 3x^2y + 3xy^2 - y^3$

   Explain or show how you came up with your answer.

	x^2	$-2xy$	y^2
x	x^3	$-2x^2y$	xy^2
$-y$	$-x^2y$	$2xy^2$	$-y^3$

$x^3 - 3x^2y + 3xy^2 - y^3$

7. Determine whether the cube of a binomial is equivalent to the sum of two cubes by exploring the following expressions:

   a. Simplify $(x + 2)^3$.

   $x^3 + 3x^2\cdot 2 + 3x\cdot(2)^2 + 2^3$

   $\boxed{x^3+6x^2+12x+8}$

   b. Simplify $x^3 + 2^3$

   $\boxed{x^3+8}$

   c. Is your answer to part a equivalent to your answer in part b? NO

   d. Simplify $(x + 2)(x^2 - 2x + 4) = x^3 - 2x^2 + \cancel{4x} + 2x^2 - \cancel{4x} + 8$

   $\boxed{x^3+8}$

   e. Is your answer to part b equivalent to your answer in part d? Yes

   f. Your answers to part b and d should be equivalent. They illustrate two more commonly used polynomial identities:

The Sum of Cubes (SOC):	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
The Difference of Cubes (DOC):	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

SAME OPPOSITE (always) POSITIVE

Mini-Lesson: Factoring Polynomials (with 3+ Degree)

2 Terms

Difference of Cubes (DOC)

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

1. $x^3 - 8$

$\sqrt[3]{x^3}$ $\sqrt[3]{8}$

x 2

a b

$(x-2)(x^2 + x \cdot 2 + 2^2)$

$\boxed{(x-2)(x^2+2x+4)}$

2. $2000x^3 - 686$

gcf: 2

$2(1000x^3 - 343)$

$\sqrt[3]{1000x^3}$ $\sqrt[3]{343}$

$10x$ 7

$\boxed{2(10x-7)(100x^2+70x+49)}$

Perfect Cube	Cube Root
1	1
8	2
27	3
64	4
125	5
216	6
343	7
512	8
729	9
1000	10

Steps & Notes

1) Factor out the GCF, if any.

2) Find the cube root of the first term and the cube root of the last term.

2) Substitute cube roots into the formulas to the left. Pay attention to the signs. SOP. *Be sure to simplify the squares, if necessary.*

Sum of Cubes (SOC)

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

3. $27x^3 + 1$

$\sqrt[3]{27x^3}$ $\sqrt[3]{1}$

$3x$ 1

a b

$(3x+1)((3x)^2 - 3x \cdot 1 + 1^2)$

$\boxed{(3x+1)(9x^2-3x+1)}$

4. $512x^3 + 125y^3$

$\sqrt[3]{512x^3}$ $\sqrt[3]{125y^3}$

$8x$ $5y$

$\boxed{(8x+5y)(64x^2-40xy+25y^2)}$

Difference of Squares (DOS) and Sum of Squares (SOS) Revisited

$$a^2 - b^2 = (a+b)(a-b)$$
$$a^2 + b^2 = (a+bi)(a-bi)$$ ← ONLY FOR QUADRATICS

5. $3x^4 - 3$

gcf: 3

$3(x^4 - 1)$

$\sqrt{x^4}$ $\sqrt{1}$

x^2 1

$3\underbrace{(x^2+1)}_{SOS}\underbrace{(x^2-1)}_{DOS}$

$\sqrt{x^2}$ $\sqrt{1}$

x 1

$\boxed{3(x+i)(x-i)(x+1)(x-1)}$

When to Use: Look for perfect squares minus or plus perfect squares. Remember variables with even exponents are all perfect squares.

Always check for GCF 1st.

Be careful using the sum of squares formula: this is strictly for ***quadratic expressions.***

6. $36x^4 - 25y^2$

$\sqrt{36x^4}$ $\sqrt{25y^2}$

$6x^2$ $5y$

$\boxed{(6x^2+5y)(6x^2-5y)}$

3 Terms

Quadratic Form (QF)

$$ax^{2n} + bx^n + c = (mx^n - p)(kx^n - q)$$

7. $x^4 - 4x^2 - 45$

-45, -4; -9/1, 5/1

$\underbrace{(x^2-9)}_{DOS}(x^2+5)$

$\sqrt{x^2}$ $\sqrt{9}$

x 3

$\boxed{(x+3)(x-3)(x^2+5)}$

8. $2x^4 + 34x^2 + 140$

gcf: 2

$2(x^4 + 17x^2 + 70)$

70, 17; 7/1, 10/1

$\boxed{2(x^2+7)(x^2+10)}$

When to Use: A polynomial expression that has three terms, one of the terms is a constant and one exponent it two times the other exponent.

Always check for GCF 1st.

Factor the expression as if it were a quadratic but then make sure that you have the correct variable exponent in the parentheses.

Check BINOMIALS for DOS, SOS, DOC and/or SOC

9. $2x^6 - x^3 - 15$

$\boxed{(x^3-3)(2x^3+5)}$

$\frac{-3}{1} = \frac{-6}{2}$ -30, -1 $\frac{5}{2}$

10. $2x^8 - 3x^4 - 35$

$\boxed{(2x^4+7)(x^4-5)}$

-70, -3; $\frac{7}{2}$ $\frac{-10}{2} = \frac{-5}{1}$

	Grouping (GRP)	Steps & Notes	Binomial Cubed (BC)
4 Terms	**When to Use:** when the expression has four terms and the degree is 3 or higher. There must be some proportionality between the pairs of terms.		$a^3+3a^2b+3ab^2+b^3=(a+b)^3$ $a^3-3a^2b+3ab^2-b^3=(a-b)^3$
	11. Factor $x^3-2x^2+5x-10$ $(x^3-2x^2)+(5x-10)$ gcf: x^2 gcf: 5 $x^2(x-2)+5(x-2)$ $\boxed{(x-2)(x^2+5)}$ **12.** Factor $x^3+2x^2-9x-18$ $(x^3+2x^2)+(-9x-18)$ gcf: x^2 gcf: -9 $x^2(x+2)-9(x+2)$ $(x+2)(x^2-9)$ DOS $\sqrt{x^2}$ x $\sqrt{9}$ 3 $\boxed{(x+2)(x+3)(x-3)}$	**Grouping** 1) Group the first two terms and the last two terms. 2) Factor the GCF out of both pairs. 3) The binomial in the parentheses should match and will be one factor and the common factors together make the second binomial. 4) If one of the factors can be factored further, keep factoring until complete. **Binomial Cubed** 1) Find the cube roots of the first and last terms. 2) Determine if the formula holds for the middle term. 3) Make a binomial of the cube roots and cube it.	**13.** Factor $x^3-15x^2+75x-125$ ✓1st & last terms are perfect cubes $\sqrt[3]{x^3}$ $\sqrt[3]{125}$ $x \leftarrow a$ $5 \leftarrow b$ ✓ Check middle terms Is $3\cdot x^2\cdot 5=15x^2$? yes (2nd) against Is $3\cdot x\cdot 5^2=75x$? yes (3rd) the terms formula $\boxed{(x-5)^3}$ **14.** Factor $8x^3+36x^2+54x+27$ $\sqrt[3]{8x^3}$ $\sqrt[3]{27}$ $2x$ 3 2nd term $(3a^2b)$ Is $3\cdot(2x)^2\cdot 3=36x^2$? yes 3rd term $(3ab^2)$ Is $3\cdot 2x\cdot 3^2=54x$? yes $\boxed{(2x+3)^3}$

Notes: If the 2nd and 4th terms have opposite signs, you will need to factor out a negative GCF from the second pair of terms.

If the binomials in the parentheses in Step 3 do not match then check your GCFs or this expression cannot be factored by grouping.