Name: ______________________ Date: ______________ Period: ________

4.5 **Factoring Quadratics**

Engage: Factoring Finesse

There is a process using box method where you can think of the product of two numbers as the product of two binomials. See below.

$31 * 37$ can be thought of as $(30+1)(30+7)$

	30	1
30	900	30
7	210	7

So $31 * 37 = 900 + 210 + 30 + 7 = 1147$

and $27 * 37$ can be thought of as $(30-3)(30+7)$

	30	-3
30	900	-90
7	210	-21

So $27 * 37 = 900 + 210 - 90 - 21 = 999$

But suppose the first number in each binomial is the same and the second number in each binomial are opposites of each other. Show your steps for finding these products. The first one has been done for you.

$38 * 42$

$(40-2)(40+2)$

	40	2
40	1600	80
-2	-80	-4

$1600 - 4 = \boxed{1596}$

$45 * 35$

$(40+5)(40-5)$

	40	+5
40	1600	200
-5	-200	-25

$1600 - 25 = \boxed{1575}$

$(22)(18)$

$(20+2)(20-2)$

	20	+2
20	400	40
-2	-40	-4

$400 - 4 = \boxed{396}$

$(x+5)(x-5)$

	x	+5
x	x^2	5x
-5	-5x	-25

$\boxed{x^2 - 25}$

Above, you computed several products of the form, $(x+y)(x-y)$, verifying that the product is always of the form $x^2 - y^2$.

1) If we choose values for *x* and *y* so that $x = y$, what happens to the product?

The product becomes zero because the same number subtracts to zero.

$(x+x)(x-x)$ | $(y+y)(y-y)$

$2x \cdot 0$ | $2y \cdot 0$

0 | 0

2) Is there another way to choose numbers for *x* and *y* so that the product of $(x+y)(x-y)$ will = 0?

If $x = -y$ the product will also equal zero since opposites add to zero.

3) In general, if the product of two numbers is zero, what must be true about one of them?

One of the numbers must be zero in order for the product to be zero

$(x+y)(x-y) = x^2 - y^2$ is called a **polynomial identity** because this statement of equality is true for all values of the variables. Polynomials in the form of $a^2 - b^2$ are called the **difference of two squares**.

In Algebra 1, you were told that you cannot factor the sum of two squares, such as $x^2 + 16$ or $x^2 + y^2$, but we know we can just not with real numbers.

4) Multiply $(x + 5i)(x - 5i)$. Describe what you see.

	x	$+5i$
x	x^2	$5xi$
$-5i$	$-5xi$	$-25i^2$

$x^2 - 25i^2$

$x^2 - 25(-1)$

$x^2 + 25$

So, the sum of squares identity is: $(x+yi)(x-yi) = x^2+y^2$

5) Jon claims that you can factor the **sum of two squares** just like the **difference of two squares**, just with i's after the constant terms. Do you agree? Why or why not?

Overall yes (if the second term is just a constant)

The i's go behind the second term in each binomial.

This relationship is another polynomial identity for the **sum of two squares**. $a^2 + b^2 = (a + bi)(a - bi)$

Now let's consider another special case when the numbers/expressions are the same.

$(x+y)^2$ $(x+y)(x+y)$	$(x-y)^2$ $(x-y)(x-y)$
$\begin{array}{c\|c\|c} & x & +y \\ \hline x & x^2 & xy \\ \hline +y & xy & y^2 \end{array}$ $\boxed{x^2 + 2xy + y^2}$	$\begin{array}{c\|c\|c} & x & -y \\ \hline x & x^2 & -xy \\ \hline -y & -xy & y^2 \end{array}$ $\boxed{x^2 - 2xy + y^2}$

These are the **perfect square trinomial** identities.

$$\mathbf{a^2 + 2ab + b^2 = (a + b)^2}$$
$$\mathbf{a^2 - 2ab + b^2 = (a - b)^2}$$

Use the identities discussed in this lesson to practice factoring on the next page. Remember to always check for greatest common factor (GCF) first.

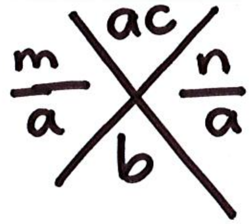
$$\text{standard form} \xrightarrow{\text{factoring}} \text{parentheses}$$
$$\text{standard form} \xleftarrow{\text{simplifying}} \text{parentheses}$$

Practice: Use the identities to factor the expressions.

	Difference of Square (DOS) $\mathbf{a^2 - b^2 = (a+b)(a-b)}$	**Sum of Squares (SOS)** $\mathbf{a^2 + b^2 = (a+bi)(a-bi)}$
2 Terms	1. $4x^2 - 9$ $\sqrt{4x^2}$ $\sqrt{9}$ $2x$ 3 $\boxed{(2x+3)(2x-3)}$	1. $81x^2 + 25$ $\sqrt{81x^2}$ $\sqrt{25}$ $9x$ 5 $\boxed{(9x+5i)(9x-5i)}$
	2. $49x^2 - 25y^2$ $\sqrt{49x^2}$ $\sqrt{25y^2}$ $7x$ $5y$ $\boxed{(7x+5y)(7x-5y)}$	2. $100x^2 + 9y^2$ $\sqrt{100x^2}$ $\sqrt{9y^2}$ $10x$ $3y$ $\boxed{(10x+3yi)(10x-3yi)}$
	3. $36x^2 - 64$ gcf: 4 $4(9x^2 - 16)$ $\sqrt{9x^2}$ $\sqrt{16}$ $3x$ 4 $\boxed{4(3x+4)(3x-4)}$	3. $14x^2 + 14$ gcf: 14 $14(x^2+1)$ $\sqrt{x^2}$ $\sqrt{1}$ x 1 $\boxed{14(x+i)(x-i)}$
	Perfect square Trinomials (PST) $\mathbf{a^2 + 2ab + b^2 = (a+b)^2}$ $\mathbf{a^2 - 2ab + b^2 = (a-b)^2}$	**Other Trinomials (TRIM)** Find two numbers that multiply to ac but add to get b. (rm = a; pq = c) $\mathbf{ax^2 + bx + c = (mx + p)(rx + q)}$
3 Terms	1. $x^2 - 18x + 81$ $\sqrt{x^2}$ $\sqrt{81}$ x 9 Does $-2 \cdot x \cdot 9 \overset{?}{=} -18x$ square roots middle term Yes $\boxed{(x-9)^2}$	1. $4x^2 - 45x + 81$ ← NOT a PST because $-2 \cdot 2x \cdot 9 \neq -45x$ 324 ac -9 -36 -45 $\frac{-9}{4}$ $\frac{-36}{4} = \frac{-9}{1}$ $\boxed{(4x-9)(x-9)}$
	2. $4x^2 + 20x + 25$ $\sqrt{4x^2}$ $\sqrt{25}$ $2x$ 5 Does $2 \cdot 2x \cdot 5 \overset{?}{=} 20x$ Yes $\boxed{(2x+5)^2}$	2. $6x^2 + 8x + 2$ gcf: 2 $2(3x^2 + 4x + 1)$ 3 3 1 4 $\frac{1}{1} = \frac{3}{3}$ $\frac{1}{3}$ $\boxed{2(x+1)(3x+1)}$
	3. $9x^2 - 42xy + 49y^2$ $\sqrt{9x^2}$ $\sqrt{49y^2}$ $3x$ $7y$ Does $-2 \cdot 3x \cdot 7y \overset{?}{=} -42xy$ Yes $\boxed{(3x-7y)^2}$	3. $12x^2 + 13x + 3$ 36 9 4 13 $\frac{2}{3} = \frac{9}{12}$ $\frac{4}{12} = \frac{1}{4}$ $\boxed{(3x+2)(4x+1)}$

To find the factors of a # in GDC:
- In calculator, go to Y=, enter #/X in Y_1
- Press 2nd graph for the table
- All non-decimal pairs are factors.

X method



m & n multiply to ac and add to b.