Name: ______________________ Date: __________ Period: ______

4.4 **Complex Numbers**

Engage: Did you Know?

Complex numbers are used in the real-world even though they have imaginary parts (pun)! Check out the different fields below.

Electrical Engineering	***Quantum Mechanics***	***Signal Processing***
Complex numbers are extensively used in electrical engineering to analyze and design circuits. They are crucial in understanding the behavior of AC (alternating current) circuits, as the voltage and current in AC circuits are represented as complex numbers. Complex numbers help engineers calculate impedance, power factor, and analyze resonance in electrical systems.	Complex numbers play a fundamental role in quantum mechanics, a branch of physics that describes the behavior of subatomic particles. Wave functions, which represent the state of particles, are complex-valued functions. Complex numbers are used to calculate probabilities, amplitudes, and describe phenomena like quantum entanglement and superposition.	Complex numbers are vital in signal processing, which is used in fields like telecommunications, audio processing, and image processing. Complex numbers allow the representation of both the amplitude and phase of a signal. They are employed in Fourier analysis, which decomposes signals into their frequency components, and in modulation techniques such as quadrature amplitude modulation (QAM).
Financial Mathematics	***Computer Graphics***	***Fluid Dynamics***
Complex numbers are utilized in financial mathematics to model and analyze financial derivatives. Complex numbers are used in the BlackScholes model for option pricing, which takes into account factors such as volatility and time to expiration. Complex analysis also aids in understanding the behavior of interest rates and analyzing complex financial data.	Complex numbers find applications in computer graphics, particularly in 2D and 3D transformations. Complex numbers can represent rotations, translations, and scaling operations. By using complex numbers, graphics software can efficiently manipulate and render objects in the virtual space.	Complex numbers are employed in fluid dynamics to analyze the behavior of fluids, such as airflow or water flow. They are used to represent the velocity potential and stream function, which help describe the flow patterns. Complex analysis techniques are applied to solve complex differential equations in fluid dynamics.

Explore: Operating with Complex Numbers

Part 1: Introduction

How do you think you would simplify the following expressions?

1. $(4+5i)+(-2+3i)$

combine like terms
4 & (-2) and $5i$ & $3i$

2. $(4+5i)-(-2+3i)$

combine like terms
4 & (2) and $5i$ & $-3i$

3. $(4+5i)(-2+3i)$

FOIL, because Multiplying two binomials
F: $4(-2)$
O: $4(3i)$
I: $5i(-2)$
L: $5i(3i)$

How are each of the original expressions above the same? How are they different?

All problems have two complex numbers, $4+5i$ and $-2+3i$

The operation between those 2 complex numbers are different for each question. #1 → addition #2 → subtraction #3 - multiplication

Part 2: Practice

Simplify the following expressions: ADDITION

4. $(3+2i)+(5+4i)$ $\boxed{8+6i}$

5. $(-2+6i)+(-3-2i)$ $\boxed{-5+4i}$

6. $(7+i)+(-7-i)$ $\boxed{0}$

7. $(4+3i)+(2-5i)$ $\boxed{6-2i}$

Describe the steps in your own words for **adding two complex numbers**.

Add the real parts together and the imaginary parts together. [combine like terms]

Simplify the following expressions: $\boxed{\text{SUBTRACTION}}$

8. $(3+2i)-(5+4i)$ **9.** $(-2+6i)-(-3-2i)$ **10.** $(7+i)-(-7-i)$ **11.** $(4+3i)-(2-5i)$

$3+2i-5-4i$ $-2+6i+3+2i$ $7+i+7+i$ $4+3i-2+5i$

$\boxed{-2-2i}$ $\boxed{1+8i}$ $\boxed{14+2i}$ $\boxed{2+8i}$

Describe the steps in your own words for **subtracting two complex numbers.**

Subtract the real parts & subtract the imaginary parts OR Distribute the minus to second complex # Then combine like terms.

Simplify the following expressions: $\boxed{\text{MULTIPLICATION}}$

12. $(3+2i)(5+4i)$ **13.** $(-2+6i)(-3-2i)$ **14.** $(7+i)(-7-i)$ **15.** $(4+3i)(2-5i)$

$15+12i+10i+8i^2$

$15+12i+10i+8(-1)$

$15+12i+10i-8$

$\boxed{7+22i}$ $\boxed{18-14i}$ $\boxed{-48-14i}$ $\boxed{23-14i}$

Describe the steps in your own words for **multiplying two complex numbers**.

FOIL (use distributive property) to get four terms

Then substitute -1 for i^2

Finally combine like terms (add real parts together & add imaginary parts together)

Part 3: Conjugates of Complex Numbers

The **complex conjugate** of $a+bi$ is $a-bi$. The real part stays the same but the imaginary part changes sign.

For the next set of problems, find the conjugate of the given complex number, then calculate the product of the complex number and its conjugate using the distributive property, and simplify the expression and identify any patterns.

The notation for the conjugate of z is z^* or $\bar{z}$.

16. Let $z = 3 + 2i$. Find the conjugate of z and calculate the product $z * z^*$. conjugate: $3-2i$

$(3+2i)(3-2i)$
$9-6i+6i-4i^2$
$9-6i+6i-4(-1)$
$9+4$
$\boxed{13}$

17. Let $w = -4 - 5i$. Find the conjugate of w and calculate the product $w * w^*$. conjugate: $-4+5i$

$(-4-5i)(-4+5i)$
$16-20i+20i-25i^2$
$16-20i+20i-25(-1)$
$16+25$
$\boxed{41}$

18. Let $x = 2i$. Find the conjugate of x and calculate the product $x * x^*$. conjugate: $-2i$

$(2i)(-2i)$
$-4i^2$
$-4(-1)$
$\boxed{4}$

19. Let $y = 4 - 7i$. Find the conjugate of y and calculate the product $y * y^*$. conjugate: $4+7i$

$(4-7i)(4+7i)$
$16+28i-28i-49i^2$
$16+28i-28i-49(-1)$
$16+49$
$\boxed{65}$

20. Let $p = -1 + 3i$. Find the conjugate of p and calculate the product $p * p^*$. conjugate: $-1-3i$

$(-1+3i)(-1-3i)$
$1+3i-3i-9i^2$
$1+3i-3i-9(-1)$
$1+9$
$\boxed{10}$

Describe the relationship between the **product of a complex number and its conjugate.** Generalize it algebraically.

The product of a complex # and its conjugate is a real number (No imaginary part)
$(a+bi)(a-bi) = a^2+b^2$

Connections: Dividing Complex Numbers

The property discovered in Part 3 is very useful when dividing complex numbers. We usually multiply the numerator and denominator of a fraction by the conjugate of the denominator.

21.
$\frac{3i}{4-5i} \cdot \frac{(4+5i)}{4+5i}$

The complex conjugate of $4-5i$ is $4+5i$

$\frac{3i(4+5i)}{4^2+5^2}$ ← distribute
← use identity $(a+bi)(a-bi) = a^2+b^2$

$\frac{12i+15i^2}{16+25}$ simplify

$\frac{12i+15(-1)}{41}$ remember $i^2 = -1$

$\frac{-15+12i}{41}$

$\underbrace{\frac{-15}{41} + \frac{12}{41}i}_{\text{standard a+bi form}}$

Worked Example for Reference

Example: Divide $(3 + 4i)$ by $(3 - 2i)$.

$$\frac{3+4i}{3-2i}$$

Find the conjugate of the denominator

$$3+2i$$

Multiply the conjugate to the numerator and denominator of the fraction.

$$\frac{3+4i}{3-2i} \cdot \frac{3+2i}{3+2i}$$

Distribute/FOIL in the numerator & use SOS rule in the denominator.

$$\frac{9+6i+12i+8i^2}{3^2+2^2}$$

Replace i^2 with – 1 and simplify the denominator

$$\frac{9+6i+12i+8(-1)}{9+4}$$

Simplify; combine like terms in the numerator and denominator separately.

$$\frac{9+18i-8}{13}$$

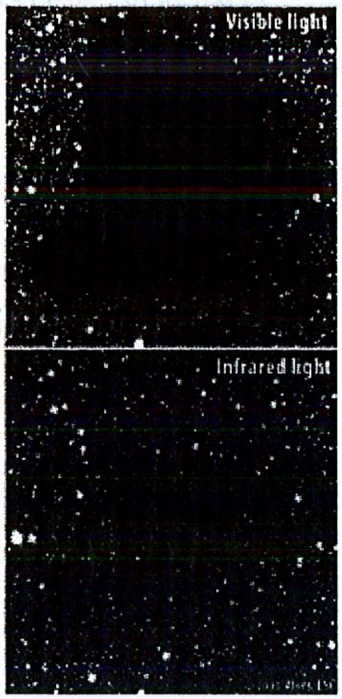
$$\frac{1+18i}{13}$$

Split the terms and reduce if possible.

$$\frac{1}{13}+\frac{18}{13}i$$

Make sure you always write the real part first and the imaginary part second.

Apply: Astronomical Complexities



For over 100 years, astronomers have been investigating how interstellar dust absorbs and reflects starlight. Too much dust and stars fade out and become invisible to optical telescopes. NASA's infrared observatories such as WISE and Spitzer, study dust grains directly through the infrared 'heat' radiation that they emit. The amount of heat radiation depends on the chemical composition of the dust grains and their reflectivity (called the albedo). Through detailed studies of the electromagnetic spectrum of dust grains, astronomers can determine their chemical composition.

The two images to the left, taken with the European Space Agency's Very Large Telescope, show the optical (top) and infrared (bottom) appearance of the interstellar dust cloud Barnard 68. They show how the dust grains behave at different wavelengths. At visible wavelengths, they make the cloud completely opaque so distant background stars cannot be seen at all. At infrared wavelengths, the dust grains absorb much less infrared light, and the cloud is nearly transparent.

$$\boldsymbol{A(m) = c \times \frac{\left|\dfrac{m^2-1}{m^2+2}\right|^2}{\mathrm{Im}\left(\dfrac{1-m^2}{m^2+2}\right)}}$$

The equation above is a mathematical model of the albedo of a dust grain, $A(m)$, as a function of its index of refraction, m, which is a complex number of the form $m = a - bi$. The denominator Im(...) is the imaginary part of the indicated complex quantity in parentheses. From your knowledge of complex numbers, answer the questions below.

An astronomer uses a dust grain composition of pure graphite for which $m = 3 - i$. What is the albedo of a 0.1-micron diameter dust grain at...

a) ultraviolet wavelengths of 0.3 microns meaning $c = 10.0$?

b) an infrared wavelength of 1 micron meaning $c = 0.1$?

NOT APPROPRIATE FOR H.S. ADVANCED ALGEBRA.