Name: ______________________ Date: __________ Period: ________

4.3 **Imaginary Numbers**

Engage: An Imaginary Story

Once upon a time, in the bustling town of Mathland, there lived a curious young girl named Lily. Lily loved exploring the world around her and solving puzzles. One sunny day, Lily came across a mysterious locked door hidden deep within the heart of the town. The door was covered in ancient symbols and seemed to hold a secret. Determined to uncover the truth, Lily embarked on a quest to find the key to unlock the door.

Her journey led her to the wise Mathemagician, who revealed the secret of the door. The Mathemagician explained, "Lily, behind this door lies a magical realm where extraordinary things happen. To unlock it, you must understand the power of complex numbers."

Intrigued, Lily asked, "What are complex numbers?"

The Mathemagician replied, "Complex numbers are like a magical bridge between real and imaginary worlds. They are numbers that go beyond what we normally see. They have both real and imaginary parts, allowing us to explore new dimensions in mathematics."

Lily's eyes widened with excitement. She realized that complex numbers were the key to unlocking the hidden door and accessing the enchanting realm of possibilities. With newfound knowledge, Lily delved into the realm of complex numbers. She discovered that they were essential in solving certain equations that couldn't be solved using only real numbers. Complex numbers had unique properties that made them incredibly powerful and useful in the world of mathematics.

As Lily explored further, she encountered complex numbers in music, art, and even nature. She saw how they brought harmony to music compositions, created intricate geometric patterns, and revealed the secrets of fractals. Complex numbers added a touch of magic to everything they touched.

With her understanding of complex numbers, Lily returned to the locked door in Mathland. Armed with the key of knowledge, she unlocked the door, and it swung open, revealing a breathtaking world beyond her imagination. In this magical realm, Lily encountered fantastical creatures, mindbending puzzles, and incredible landscapes. She realized that without complex numbers, this world would remain hidden and inaccessible.

Filled with awe and gratitude, Lily returned to Mathland and shared her story with the townspeople. They marveled at the power of complex numbers and how they unlocked a world of possibilities. Lily became a hero, inspiring future mathematicians, and explorers to embrace the wonders of complex numbers and continue the quest for knowledge. And so, the legend of Lily, the girl who discovered the magic of complex numbers, spread far and wide, reminding everyone that behind every locked door lies a world waiting to be explored with the power of imagination and mathematics.

Explore: iPatterns

Part 1: Rewrite the following expressions in simplest terms writing out each step and determining the pattern. Two examples are given to the right.

$\sqrt{4} * \sqrt{4}$	$(\sqrt{25})^2$
$2 * 2$	5^2
4	25

1. $\sqrt{16} * \sqrt{16}$ $4 \cdot 4$ 16	2. $\sqrt{100} * \sqrt{100}$ $10 \cdot 10$ 100	3. $\sqrt{81} * \sqrt{81}$ $9 \cdot 9$ 81	4. $(\sqrt{9})^2$ 3^2 9	5. $(\sqrt{36})^2$ 6^2 36
6. $\sqrt{x^2} * \sqrt{x^2}$ $x \cdot x$ x^2	7. $\sqrt{9x^4} * \sqrt{9x^4}$ $3x^2 \cdot 3x^2$ $9x^4$	8. $\sqrt{-1} * \sqrt{-1}$ $i \cdot i = i^2$ -1	9. $\sqrt{i} * \sqrt{i}$ $\sqrt{i^2}$ i	10. $\sqrt{i^2} * \sqrt{i^2}$ $i \cdot i$ i^2

11. What do you notice about the expressions in the table?

The square root of a expression times itself is equal to the radicand.

Part 2: For this task, the letter i denotes the imaginary unit, that is $\boxed{i = \sqrt{-1}}$. So this means $i^2 =$ -1.

12. For each integer k from 0 to 8, write i^k in the form $a + bi$.

i^0	i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8
1	i	-1	$-i$	1	i	-1	$-i$	1

13. Describe the pattern you observe.

The powers are cyclic going from $i \to -1 \to -i \to 1$ and back to i.

14. Write your observation algebraically.

odd exponents one more than a mult of 4 $\to i$
even exponents Not multiples of 4 $\to -1$
odd exponents THREE more than a mult. of 4 $\to -i$
even exponents that are multiples of 4 $\to 1$

15. Simplify i^{195}.

$\frac{195}{4} = 48.75$ or $48\frac{3}{4}$ 3 more than a multiple of 4

$i^{195} = \boxed{-i}$

16. In your own words, explain why $i^2 = -1$.

$i^2 = -1$ because $i = \sqrt{-1}$ and $\sqrt{-1} \cdot \sqrt{-1} = -1$

Part 3: Answer the questions below.

The iChant

I Won(one),
I Won(one)
(Negatives in the Middle)

i
-1
$-i$
1

17. Write each of the following expressions in the form $a + bi$.

a. $i^2 + i + 1 = -1 + i + 1 = \boxed{i}$ $[0+i]$

b. $i^3 + \underbrace{i^2 + i + 1}_{\text{part a}} = -i + i = \boxed{0}$ $[0+0i]$

c. $i^4 + \underbrace{i^3 + i^2 + i + 1}_{\text{part b}} = 1 + 0 = \boxed{1}$ $[1+0i]$

d. $i^5 + \underbrace{i^4 + i^3 + i^2 + i + 1}_{\text{part c}} = i + 1 = \boxed{1+i}$

e. $i^6 + \underbrace{i^5 + i^4 + i^3 + i^2 + i + 1}_{\text{part d}} = -1 + 1 + i = \boxed{i}$

f. $i^7 + \underbrace{i^6 + i^5 + i^4 + i^3 + i^2 + i + 1}_{\text{part e}} = -i + i = \boxed{0}$

g. $i^8 + \underbrace{i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1}_{\text{part f}} = 1 + 0 = \boxed{1}$

18. Describe the pattern you observe, and algebraically prove your observation.

The sums of $1 + i + i^2 \ldots i^n$ follow a cyclic pattern of: 1, $1+i$, i, 0
If n is a multiple of 4, the sum is 1. If n is 3 more than a multiple of 4, the sum is 0. If n is even but not a multiple of 4 the sum is i. If n is 1 more than a multiple of 4, the sum is $1+i$.

19. Compute $i^{195} + i^{194} + \cdots + i^2 + i + 1$.

$= \boxed{0}$

since 195 is 3 more than a multiple of 4 (from #15).

Practice: $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$

To summarize, an imaginary number is any number whose square is negative. This means an imaginary number is any number of the form bi where b is a real number and $b \neq 0$. The *square root* of any negative number results in an imaginary number.

Write the following square roots as imaginary numbers:

a) $\sqrt{-4}$ $=\sqrt{-1}\cdot\sqrt{4}$
$=i\cdot 2$
$=\boxed{2i}$

b) $\sqrt{-3}$ $=\sqrt{-1}\cdot\sqrt{3}$
$=\boxed{i\sqrt{3}}$

c) $\sqrt{-24}$ $=\sqrt{-1}\cdot\sqrt{24}$
$=i\cdot 2\sqrt{6}$
$=\boxed{2i\sqrt{6}}$

24
4 6
2 2 2 3

d) $-2\sqrt{-28}$ $=-2\sqrt{-1}\cdot\sqrt{28}$
$=-2\cdot i\cdot 2\sqrt{7}$
$=\boxed{-4i\sqrt{7}}$

28
4 7
2 2

Notation Tip:

Put the "i" in front of the radical if there is still a square root in the number.

Put the "i" after the number if there is no square root in the number.

Using the iCycle, evaluate the following powers:

e) i^{24} $=\boxed{1}$

24 is a Multiple of 4

f) i^{47} $=\boxed{-i}$

47 is 3 more than a Multiple of 4

g) i^{1001} $=\boxed{i}$

1001 is 1 more than a Multiple of 4

h) i^{38} $=\boxed{-1}$

38 is even but not a Multiple of 4

Simplify the expressions below. Note: remember to evaluate the powers first and then combine like terms if possible.

j) $i^{26}+i^{6}$ $=(-1)+(-1)$
$=\boxed{-2}$

k) $3i^{39}-i^{8}$ $=3(-i)-1$
$=-3i-1$
$=\boxed{-1-3i}$ ← written in standard a+bi form

You have noticed that we have written our answers in the form $a+bi$ on the previous page. This is the general form of **complex numbers**. The set of complex numbers are numbers that have a real part and imaginary part. This is the largest set of numbers to date.

For the complex numbers below, circle the real part and underline the imaginary part. $a+\underline{bi}$

l) $3+\underline{2i}$ m) $5-\underline{6i}$ n) $\underline{8i}+3$ o) $\underline{-12i}-5$

In the next lesson, we will perform operations with complex numbers.