

## 4.2 The Forms of a Quadratic Function

### Engage: Will It Hoop?

As you are working through the Desmos Activity, Will it Hoop?, Record your thoughts for each round.

Round 1 is just watching the clip of half the path of the shot. [Predictions]

Round 2 is when you fit a quadratic model to the path of the shot. [Parabolas]

Round 3 is the verification of watching the video of the complete shot. [Verify]

Shot #	Round 1 In/Out	Round 2 In/Out	Round 3 In/Out	Summarize your thoughts from the class results
1			IN	The predictions yielded an average of about 3.3 correct. The parabolas yielded an average of about 5 <del>correct</del> correct. Students were better at making predictions when they could make parabolas.
2			OUT	
3			IN	
4			OUT	
5			IN	
6			OUT	
7			IN	

*Answers vary for Round 1 & 2*

### Explore: Part 1 - Match My Parabola

After completing the Match My Parabola activity on Desmos, answer the following questions:

1. Share one interesting insight or connection you made during the activity that you found particularly intriguing or surprising.

*Answers will vary.*

2. Which form was easiest for you to use and understand (standard, vertex or factored) and why?

*Answers will vary.*

3. Which form was most difficult for you to use and understand (standard, vertex, or factored) and why?

*Answers will vary.*

Part 2 - Quadratic Transformation Trek

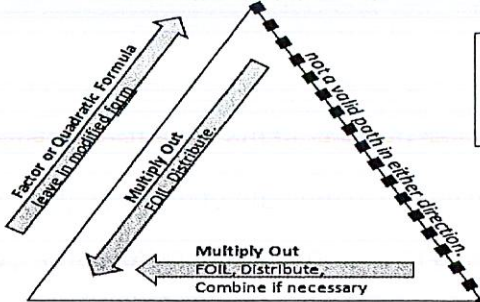
Example(s):  
 Intercept to Standard  
 $y = 2(x+3)(x-1)$   
 $y = 2(x^2 - x + 3x - 3)$   
 $y = 2(x^2 + 2x - 3)$   
 $y = 2x^2 + 4x - 6$   
 Standard to Intercept  
 $y = x^2 - 3x - 15$   
 $y = (x-5)(x+2)$

**Standard Form**  
 $y = ax^2 + bx + c$

Example(s):  
 Vertex to standard  
 $y = 2(x+3)^2 - 22$   
 $y = 2(x^2 + 6x + 9) - 22$   
 $y = 2x^2 + 12x + 18 - 22$   
 $y = 2x^2 + 12x - 4$   $\left\{ \begin{array}{l} a=2 \\ b=12 \\ c=-4 \end{array} \right.$   
 Standard to vertex (Level 1)  
 $y = 3x^2 + 24x + 45$   
 $y = 3(x^2 + 8x + 15)$   
 $y = 3(x+2)^2 - 3$

- Notes:
- "a" is the same in all three forms.
  - The numbers in the parentheses always switch signs when writing as points.
  - "a" is the vertical distance one unit to the right of the vertex.

**Factored Form\***  
 $y = a(x-p)(x-q)$



\*This factored form does not slide the denominators in front of x when performing slip-slide-divide. It is more of an Intercept form instead of factored form.

**Vertex Form**  
 $y = a(x-h)^2 + k$

Multiple Methods  
 See below

- Level 1: Use calculator to find maximum/minimum, this is the vertex (h, k). Substitute into formula.  
 Level 2: Find axis of symmetry (also h). Then substitute h into function to find k.  
 $x = -\frac{b}{2a} = h$  and  $f(-\frac{b}{2a}) = k$   
 Level 3: Complete the square (using the complete process).

Example(s):  
 Standard to vertex (Level 2)  
 $y = x^2 + 12x + 32$   
 $h = -\frac{12}{2(1)} = -6$   
 $k = (-6)^2 + 12(-6) + 32 = -4$   
 $y = (x+6)^2 - 4$   
 Standard to vertex (Level 3)  
 $y = x^2 + 12x + 32$   
 $y - 32 = x^2 + 12x$   
 $y - 32 + 36 = x^2 + 12x + 36$   
 $y + 4 = (x+6)^2 + 36$   
 $y + 4 = (x+6)^2$   
 $y = (x+6)^2 - 4$

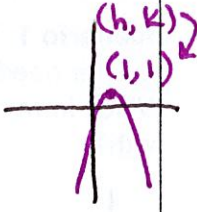
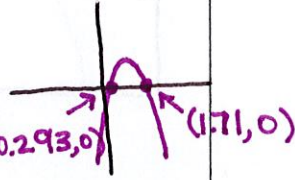
Analyze each equation and convert it to the other two forms.

	Standard Form	Vertex Form	Factored Form
A	$y = -2x^2 + 4x - 1$	$y = -2(x-1)^2 + 1$	NOT factorable over the rational numbers.
B	$y = x^2 - 4x + 7$	$y = (x-2)^2 + 3$	Not factorable over the real numbers
C	$y = x^2 - 2x - 3$	$y = (x-1)^2 - 4$	$y = (x+1)(x-3)$
D	$y = -0.5x^2 - 2x + 5$	$y = -0.5(x+2)^2 + 7$	Not factorable over the rational numbers
E	$y = x^2 + 8x + 10$	$y = (x+4)^2 - 6$	Not factorable over the rational numbers
F	$y = x^2 + x - 2$	$y = (x+0.5)^2 - 2.25$	$y = (x+2)(x-1)$

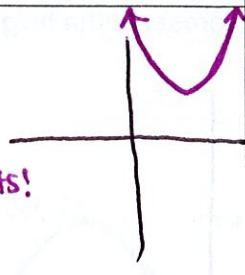
A & B Notes are on next page.

Use the next page to show the work for converting A and B.

Example A: Form given:  $y = -2x^2 + 4x - 1$

	Algebraically (no calculator)	Graphically (with calculator)
Change to vertex form	$y = -2x^2 + 4x - 1$ $a = -2 \quad b = 4 \quad c = -1$ <p>① Find AOS ② substitute into eq to find y/k.</p> $x = \frac{-b}{2a} \quad y = -2(1)^2 + 4(1) - 1$ $x = \frac{-(4)}{2(-2)} \quad y = -2 + 4 - 1 = 1$ $x = 1 \quad \text{Vertex: } (1, 1) \rightarrow (h, k)$ $y = -2(x-1)^2 + 1$	<ul style="list-style-type: none"> <li>Enter eq in <math>Y=</math></li> <li><math>Y_1 = -2x^2 + 4x - 1</math></li> <li>Press <b>graph</b></li> <li>Press <b>2ND trace</b></li> <li>4: maximum</li> </ul> $y = -2(x-1)^2 + 1$ 
Change to factored form	<p>attempt to factor</p> <del> <math display="block">\begin{matrix} 2 \\ \times \\ 4 \\ b \end{matrix}</math> </del> <p>Find 2 integers that multiply to 2 and add to 4. NOT POSSIBLE!</p> <p>— NOT FACTORABLE OVER THE RATIONAL NUMBERS —  <math>\{ \text{Intercept form is for rational } p \ \&amp; \ q \}</math></p>	<ul style="list-style-type: none"> <li>Enter 0 in <math>Y_2</math></li> <li>Press <b>2nd trace</b></li> <li>5: intersect <math>(0.293, 0)</math></li> </ul> <p>X-Intercepts are <u>irrational</u></p> 

Example B: Form given:  $y = (x-2)^2 + 3$

	Algebraically (no calculator)	Graphically (with calculator)
Change to standard form	<p>Multiply it out.</p> $y = (x-2)^2 + 3$ $y = (x-2)(x-2) + 3 \quad \text{FOIL}$ $y = x^2 - 2x - 2x + 4 + 3 \quad \text{combine like terms}$ $y = x^2 - 4x + 7$	<p>NOT IDEAL!</p>
Change to factored form	<p>① must convert to standard form 1st to factor it.</p> <p>② attempt to factor</p> <del> <math display="block">\begin{matrix} 7 \\ \times \\ -4 \\ b \end{matrix}</math> </del> <p>Find 2 integers that multiply to 7 and add to -4. NOT POSSIBLE!</p>	<ul style="list-style-type: none"> <li>Enter 0 in <math>Y_2</math></li> <li>This graph has No x-intercepts!</li> </ul> <p>NOT FACTORABLE OVER THE REAL NUMBERS</p> 

**Note:** Changing from Factored form to the other forms is the easiest algebraically. Multiply everything out to go to standard form and then find the vertex to put it in vertex form.