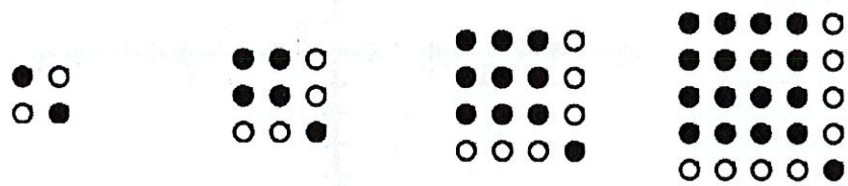
Name: ______________________________ Date: ______________ Period: ________

4.11 Graphs & Characteristics of Polynomials

Engage: Polynomial Patterns

$n = 1$ $n = 2$ $n = 3$ $n = 4$



a) For the diagram to the left, what do you notice and wonder?

Answers will vary.

b) Describe how you see the number of black dots changing or the number of white dots changing. Choose one color to focus on.

Black Dots

n	1	2	3	4
# of dots	2	5	10	17

+3 +5 +7

increase by consecutive odd #'s each time starting with 3

White Dots

n	1	2	3	4
# of dots	2	4	6	8

+2 +2 +2

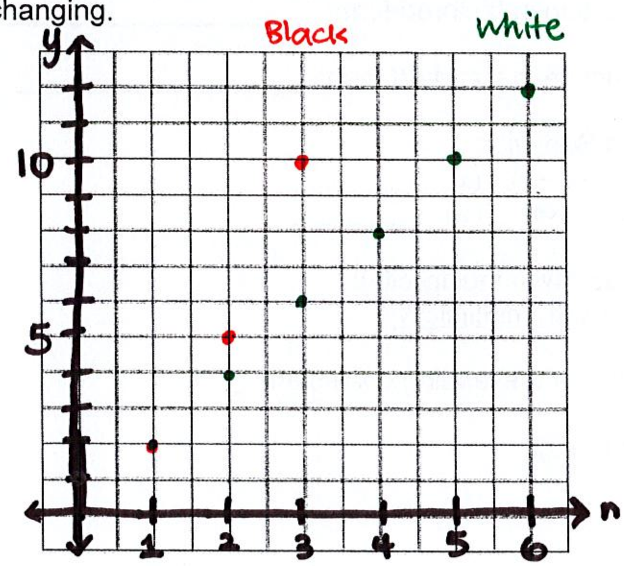
increase by 2

c) Write an equation to model how the dots are changing for the color you chose in part b).

Black : $y = n^2 + 1$

White : $y = 2n$

d) Create a graph to model how the dots are changing.



e) How could the graph be useful?

—

f) What is a real-life scenario that these images might be representing?

Answers will vary.

Explore: Puzzling over Polynomials

Learning Focus: Combine pieces of information about polynomials to write equations and graph them. Identify features of polynomials from equations and graphs.

Each of these polynomial puzzles given contain a few pieces of information. Your job is to use that information to complete the puzzle. Occasionally, you may find a missing piece that you can fill in yourself. For instance, although some of the roots are given, you may decide that there are others that you can fill in. When you need to graph a function, imagine what it will look like before using technology. Then use technology to graph the function and see how close your idea was to the actual function.

As you are working through the problem, pause and reflect after each one to answer the question: **What are the characteristics of the function that you knew from just the equation that was given?**

1)

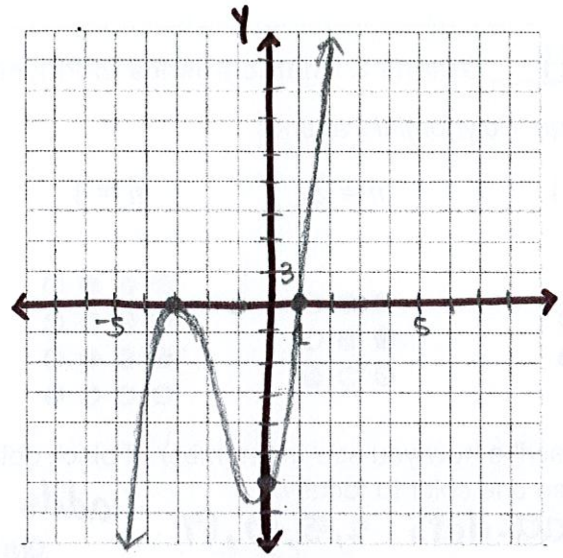
Function in factored form: $f(x) = 2(x-1)(x+3)^2$

End Behavior:
As $x \to -\infty, f(x) \to$ $-\infty$
As $x \to \infty, \quad f(x) \to$ ∞

Roots (with multiplicity):
1, −3 w/ mult. 2

Value of the leading coefficient: 2

Domain: all real #'s $(-\infty,\infty)$ Range: all real numbers $(-\infty,\infty)$



2)

Function in factored form: $f(x) = -2(x+2)(x-1)^2$

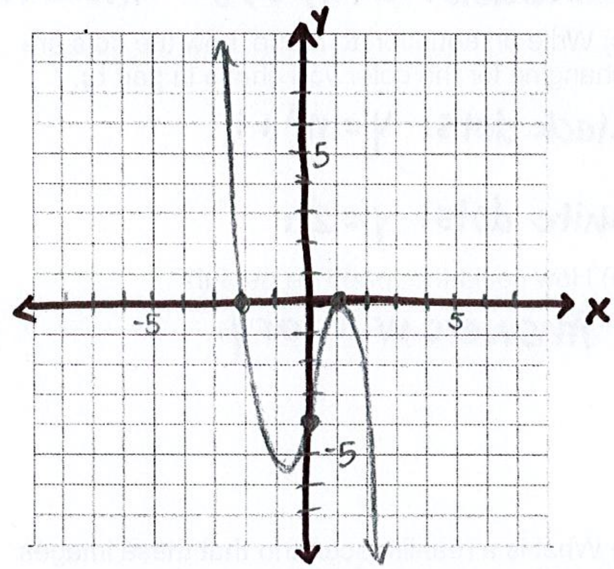
Function in standard form: $f(x) = -2x^3+6x-4$

End Behavior:
As $x \to -\infty, f(x) \to$ ∞
As $x \to \infty, \quad f(x) \to$ $-\infty$

Roots (with multiplicity):
-2, 1 with multiplicity 2

Value of the leading coefficient: -2

Degree: 3



3)

Function in factored form: $f(x) = -x^2(x-3)(x+1)^2$

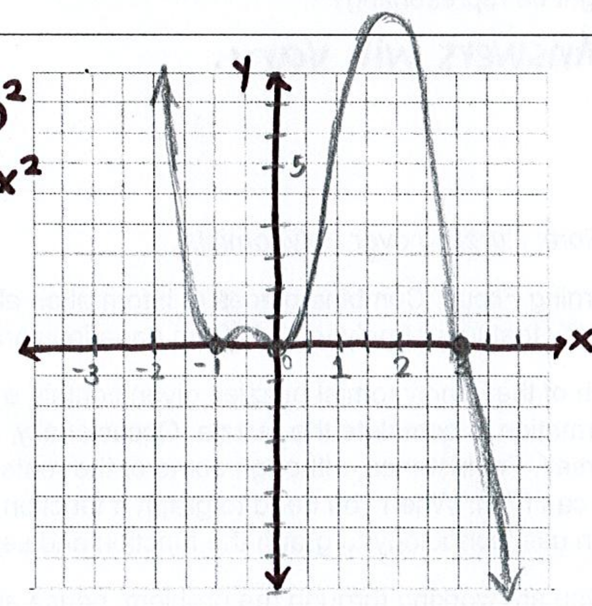
Function in standard form: $f(x) = -x^5+x^4+5x^3+3x^2$

End Behavior:
As $x \to -\infty, f(x) \to$ ∞
As $x \to \infty, \quad f(x) \to$ $-\infty$

Roots (with multiplicity):
3, -1 with multiplicity 2, 0 with multiplicity 2

Value of the leading coefficient: -1

Domain: all real #'s $(-\infty,\infty)$ Range: all real #'s $(-\infty,\infty)$



4)

Function in factored form: $f(x) = (x-2)^2(x+2)^2$

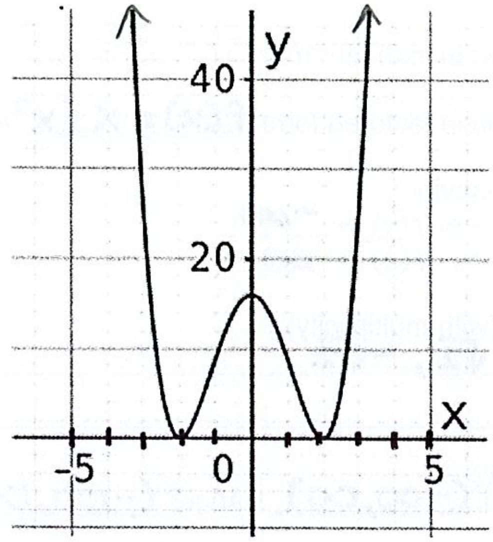
End Behavior:
As $x \to -\infty, f(x) \to$ ∞
As $x \to \infty, \quad f(x) \to$ ∞

Roots (with multiplicity):
-2 with mult. 2; 2 with mult. 2

Value of the leading coefficient: 1

Domain: $x \in (-\infty, \infty)$ Range: $y \in [0, \infty)$

Other: $f(0) = 16$



5)

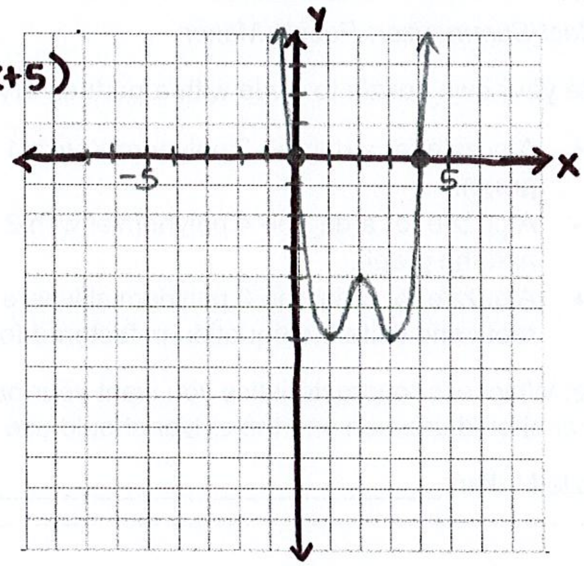
Function in factored form: $f(x) = x(x-4)(x^2-4x+5)$

End Behavior:
As $x \to -\infty, f(x) \to$ ∞
As $x \to \infty, \quad f(x) \to$ ∞

Roots (with multiplicity):
$2+i$, 4, 0, $2-i$

Value of the leading coefficient: 1

Degree: 4



6)

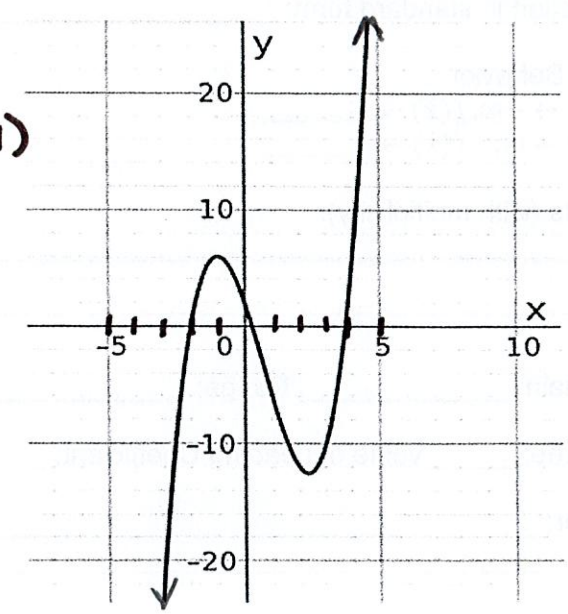
Function in standard form: $f(x) = x^3 - 2x^2 - 7x + 2$

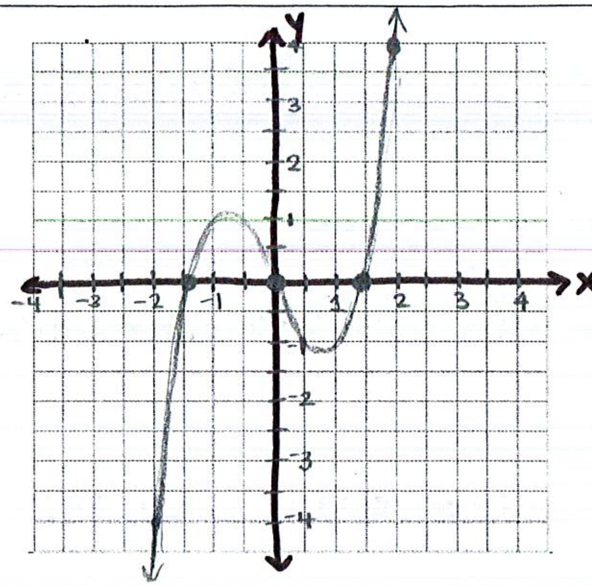
Function in factored form: $f(x) = (x+2)(x^2-4x+1)$

End Behavior:
As $x \to -\infty, f(x) \to$ $-\infty$
As $x \to \infty, \quad f(x) \to$ ∞

Roots (with multiplicity):
-2, $2+\sqrt{3}$, $2-\sqrt{3}$

Domain: $x \in (-\infty, \infty)$ Range: $y \in (-\infty, \infty)$



7) Function in standard form: $f(x) = x^3 - 2x$ Function in factored form: $f(x) = x(x^2-2)$ End Behavior: As $x \to -\infty, f(x) \to$ $-\infty$ As $x \to \infty, \quad f(x) \to$ ∞ Roots (with multiplicity): $0, \sqrt{2}, -\sqrt{2}$ Domain: $x \in (-\infty, \infty)$ Range: $y \in (-\infty, \infty)$	

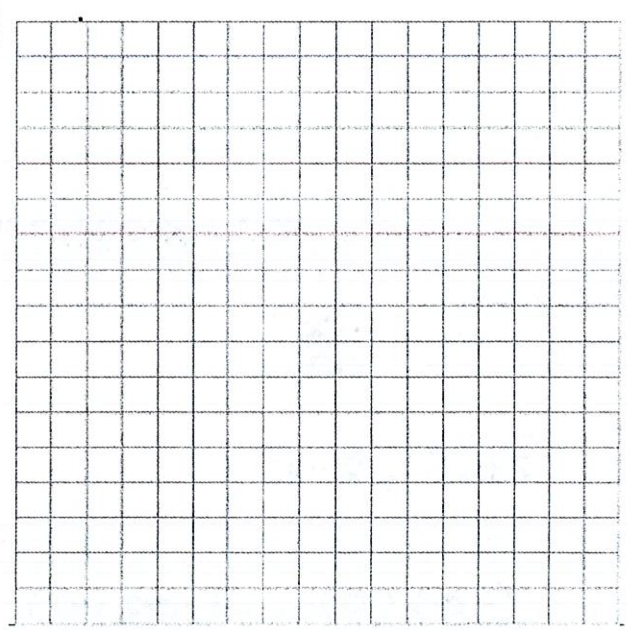
Reflect/Summarizer: Puzzle Maker

Write your own puzzle to trade with a partner. Try to come up with one of these for your partner:

- A puzzle for a degree 3 polynomial with 1 real root. The solver needs to find the equation and the graph.
- A puzzle for a degree 4 polynomial with 2 complex roots. The solver needs to find the equation and the graph.
- A puzzle for a degree 3 polynomial where the solver is given the graph and needs to find the roots and write the equation in factored form.

Note: Whichever characteristics you want your partner to ignore, draw a thick line through it. The puzzle maker should use pen and the solver should use pencil.

Puzzle Maker: ______________________ Solver: ______________________

Function in factored form: ______________ Function in standard form: ______________ End Behavior: As $x \to -\infty, f(x) \to$ ______ As $x \to \infty, \quad f(x) \to$ ______ Roots (with multiplicity): ______________________________ ______________________________ Domain: __________ Range: __________ Degree: _____ Value of Leading Coefficient: _____ Other: ______________________________	

Graphs & Characteristics of Polynomials

① $f(x) = 2x^3 + \ldots$ ← 1ST TERM   POS & ODD 

$f(0) = 2(-1)(3)^2$
$= -18$ ← y-intercept

② $f(x) = -2(x+2)\underbrace{(x-1)^2}_{\text{PST}}$ ← factored form

$f(x) = -2(x+2)(x^2-2x+1)$
$f(x) = -2(x^3-2x^2+x+2x^2-4x+2)$
$f(x) = -2(x^3-3x+2)$
$f(x) = -2x^3+6x-4$

$f(0) = -2(0)^3+6(0)-4$
$f(0) = -4$ ← y-intercept

1ST TERM: $-2x^3$ NEG & ODD 

③ $f(x) = -x^2(x-3)\underbrace{(x+1)^2}_{\text{PST}}$ ← factored form

$f(x) = -x^2(x-3)(x^2+2x+1)$
$= -x^2(x^3+2x^2+x-3x^2-6x-3)$
$= -x^2(x^3-x^2-5x-3)$
$f(x) = -x^5+x^4+5x^3+3x^2$ ← standard form

1ST TERM: $-x^5$ NEG & ODD 

④ $f(x) = (x-2)^2(x+2)^2$ ← factored form

$f(x) = (x^2-4x+4)(x^2+4x+4)$
$f(x) = x^4-8x^2+16$ ← standard form

	x^2	$4x$	4
x^2	x^4	$4x^3$	$4x^2$
$-4x$	$-4x^3$	$-16x^2$	$-16x$
$+4$	$4x^2$	$16x$	16

⑤ 1ST TERM: x^4 (given)   POS EVEN 

ROOTS: $2+i, 4, 0, 2-i$
↑ implied by CCT

$$f(x) = x(x-4)[x-(2+i)][x-(2-i)]$$
$$= x(x-4)[(x-2)-i][(x-2)+i]$$
$$= x(x-4)[(x-2)^2+1]$$
$$= x(x-4)[x^2-4x+4+1]$$
$$= x(x-4)(x^2-4x+5) \quad \leftarrow \text{factored form}$$

Extra →
$$x(x-4)(x^2-4x+5)$$
$$(x^2-4x)(x^2-4x+5)$$
$$x^4-4x^3+5x^2-4x^3+16x^2-20x$$
$$f(x) = x^4-8x^3+21x^2-20x \quad \leftarrow \text{standard form}$$

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6. Since -2 is a root → $x+2$ is a factor

let's use long division.

$$\begin{array}{r l}
 & x^2 - 4x + 1 \\
x+2 \,\big) & x^3 - 2x^2 - 7x + 2 \\
 & -(x^3 + 2x^2) \\
\hline
 & -4x^2 - 7x \\
 & -(-4x^2 - 8x) \\
\hline
 & x + 2 \\
 & -(x + 2) \\
\hline
 & 0
\end{array}$$

$$f(x) = (x+2)\underbrace{(x^2-4x+1)}_{\text{cannot be factored over the rational numbers}} \quad \leftarrow \text{factored form}$$

To find the other roots, solve the equation $x^2-4x+1=0$

$$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \frac{4}{2} \pm \frac{2\sqrt{3}}{2}$$

$x = 2+\sqrt{3}$ and $x = 2-\sqrt{3}$

≈ 3.73 $\quad\quad$ ≈ 0.268

(7) 1ST TERM: x^3 POS & ODD 

$f(x) = x^3 - 2x$

$f(x) = x(x^2-2)$ ← factored form

To find roots, solve $x(x^2-2)=0$

$x = 0$ or $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

roots: $0, \sqrt{2}, -\sqrt{2}$