

### 2.7 Operations with Functions

You can add, subtract, multiply and divide functions together. You can also add, subtract, multiply or divide outputs of functions.

Operations	Notations	What to Do
Add	$(f + g)(x)$ $f(x) + g(x)$	COMBINE like terms
Subtract	$(f - g)(x)$ $f(x) - g(x)$	write the difference distribute the minus combine like terms
Multiply	$g \circ f$ $gf$ $(g \cdot f)(x)$ $g(x) \cdot f(x)$	distribute or FOIL depending on # of terms.
Divide	$(g \div f)(x)$ $\left(\frac{g}{f}\right)(x)$ $\frac{g(x)}{f(x)}$	write functions as a <u>fraction</u> .

Suppose  $f(x) = 2x + 3$  and  $g(x) = -4x^2 + 12$  and  $h(x) = \frac{6}{x}$ . Find the following operations.

<p>1. <math>f(x) + g(x)</math></p> $\underline{(2x+3)} + \underline{(-4x^2+12)}$ $2x + 15 - 4x^2$	<p>2. <math>g(x) - f(x)</math></p> $(-4x^2+12) - (2x+3)$ $-4x^2+12 - 2x - 3$ $-4x^2 + 9 - 2x$	<p>3. <math>(h \circ g)(x)</math></p> $h(x) \cdot g(x)$ $\frac{6}{x} \cdot (-4x^2+12)$ $\frac{6}{x} \cdot \frac{-4x^2}{1} + \frac{6}{x} \cdot \frac{12}{1}$ $-24x + \frac{72}{x}$
<p>4.</p> $\frac{g(x)}{f(x)}$ $\frac{-4x^2+12}{2x+3}$	<p>5. <math>(f - g)(x)</math></p> $f(x) - g(x)$ $(2x+3) - (-4x^2+12)$ $2x+3 + 4x^2 - 12$ $4x^2 + 2x - 9$	<p>6. <math>f(x) \cdot g(x)</math></p> $(2x+3)(-4x^2+12)$ $-8x^3 + 24x - 12x^2 + 36$

Note: These problems must be done **without** a calculator.

Sometimes, the problems have a number where  $x$  is and that means that they want you to add, subtract, multiply, or divide the outputs of those functions for those values of  $x$ . **You should evaluate each function first and then perform the operations** with those outputs.

Suppose  $f(x) = x^2 + 1$  and  $g(x) = -x + 5$  and  $h(x) = \sqrt{x}$

<p>7. Find <math>f(2) \cdot g(2)</math></p> <p><math>f(2) = 2^2 + 1</math>    <math>g(2) = -2 + 5</math>  <math>f(2) = 5</math>        <math>g(2) = 3</math></p> <p><math>f(2) \cdot g(2)</math>  <math>5 \cdot 3</math>  <span style="border: 1px solid black; padding: 2px;">15</span></p>	<p>8. <math>(h + g)(4)</math></p> <p><math>h(4) + g(4)</math></p> <p><math>h(4) = \sqrt{4}</math>        <math>g(4) = -4 + 5</math>  <math>h(4) = 2</math>         <math>g(4) = 1</math></p> <p><math>2 + 1</math>  <span style="border: 1px solid black; padding: 2px;">3</span></p>	<p>9. <math>(\frac{h}{g})(9)</math>    <math>\frac{h(9)}{g(9)}</math></p> <p><math>h(9) = \sqrt{9}</math>        <math>g(9) = -9 + 5</math>  <math>h(9) = 3</math>         <math>g(9) = -4</math></p> <p><span style="border: 1px solid black; padding: 2px;">\frac{3}{-4}</span></p>
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Use your calculator to check. ALPHA TRACE ~~NUMBER~~ for function shortcut. 1:  $y_1$   
 2:  $y_2$   
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### Composition of Functions

This is the last operation where you put one function inside of another.

Notations:  $g(f(x))$  or  $(g \circ f)(x)$  Say "g of f of x"

Please know that order matters, so, 90% of the time  $g(f(x)) \neq f(g(x))$

<p>Suppose <math>f(x) = x^2</math> and <math>g(x) = 2x + 3</math></p>	
<p>Find <math>f(g(x))</math></p> <p><math>f(2x+3) = (2x+3)^2</math>  <math>= (2x+3)(2x+3)</math>  <math>= 4x^2 + 6x + 6x + 9</math>  <math>f(g(x)) = 4x^2 + 12x + 9</math></p>	<p>Find <math>g(f(x))</math></p> <p><math>g(x^2) = 2x^2 + 3</math>  <math>g(f(x)) = 2x^2 + 3</math></p>
<p>Suppose <math>f(x) = -x + 5</math> and <math>g(x) = 3 - 2x</math></p>	
<p>Find <math>f(g(x))</math></p> <p><math>f(3-2x) = -1(3-2x) + 5</math>  <math>= -3 + 2x + 5</math>  <math>f(g(x)) = 2 + 2x</math></p>	<p>Find <math>g(f(x))</math></p> <p><math>g(-x+5) = 3 - 2(-x+5)</math>  <math>= 3 + 2x - 10</math>  <math>g(f(x)) = 2x - 7</math></p>

Now with input values...

<p>Suppose <math>f(x) = 2x + 1</math> and <math>g(x) = x^2</math></p>			
<p>a) Find <math>f(g(3))</math></p> <p><math>g(3) = 3^2</math>  <math>= 9</math></p> <p><math>f(9) = 2(9) + 1</math>  <math>= 19</math></p>	<p>b) Find <math>g(f(-1))</math></p> <p><math>f(-1) = 2(-1) + 1</math>  <math>= -2 + 1</math>  <math>= -1</math></p> <p><math>g(-1) = (-1)^2</math>  <math>= 1</math></p>	<p>c) <math>g(g(2))</math></p> <p><math>g(2) = 2^2</math>  <math>= 4</math></p> <p><math>g(4) = 4^2</math>  <math>= 16</math></p>	<p>d) <math>g(f(5))</math></p> <p><math>f(5) = 2(5) + 1</math>  <math>= 11</math></p> <p><math>g(11) = 11^2</math>  <math>= 121</math></p>

Don't forget to check your answers with the calculator when you have input values.