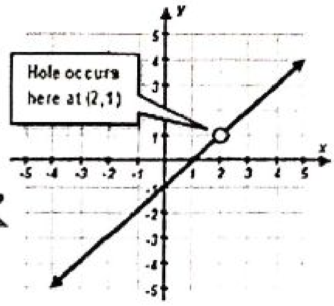
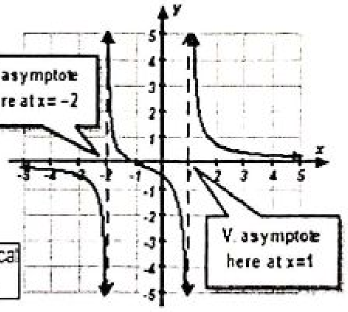
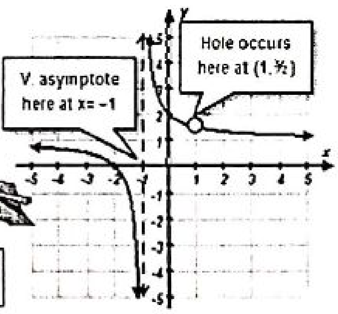
KEY

7.3 – Graphing Rational Functions

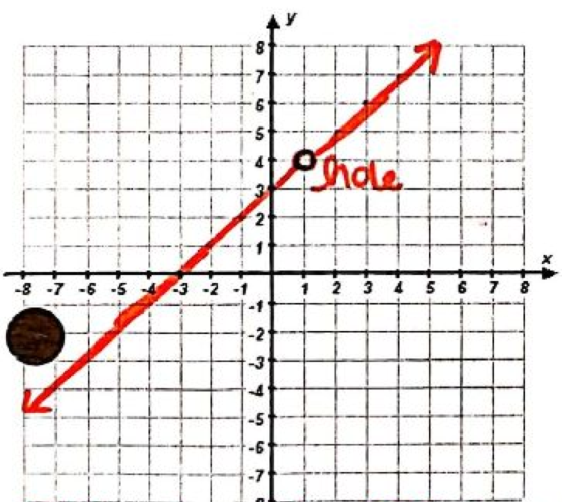
Name:

Characteristic	Description	Example
(Point Discontinuity)	A hole **usually** occurs in the graph of a rational function when a linear factor in the numerator and denominator "divide out". The result is the same as the graph of the simplified function but with a missing point in the graph.	**Graph:** $f(x) = \frac{x^2-3x+2}{x-2}$ **Simplifies:** $f(x) = \frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}}$ This will create a hole at $x = 2$. 
Vertical Asymptote *(Infinite Discontinuity)*	A vertical asymptote occurs any time a linear factor of the denominator doesn't "divide out" with a factor in the numerator.	**Graph:** $f(x) = \frac{x+1}{x^2+x-2}$ **Simplifies:** $f(x) = \frac{(x+1)}{(x+2)(x-1)}$ This will create a vertical asymptote at $x = -2$. This will create a vertical asymptote at $x = 1$. 
Vertical Asymptote & Hole	To have both a hole and a vertical asymptote the rational function must have at least one linear factor that divides out and one linear factor that does not.	**Graph:** $f(x) = \frac{x^2+x-2}{x^2-1}$ **Simplifies:** $f(x) = \frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$ This will create a vertical asymptote at $x = -1$. This will create hole at $x = 1$. 

Sketch a graph of the following rational functions. Label any holes or vertical asymptotes. Use your calculator for additional assistance.

1. $f(x) = \frac{x^2+2x-3}{x-1}$ $\frac{(x+3)(x-1)}{x-1}$

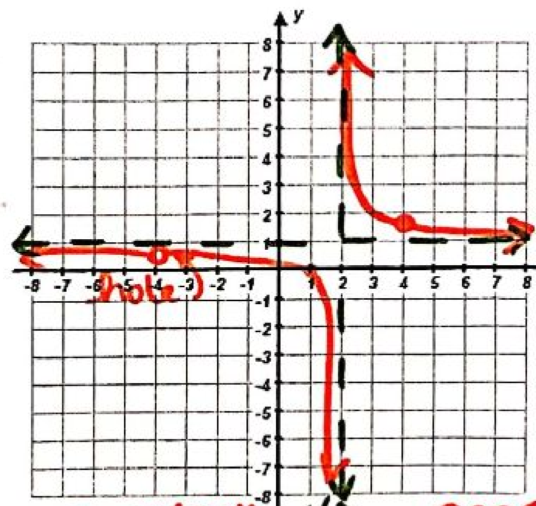
VA: NONE
HA: NONE
HOLE: (1,4)



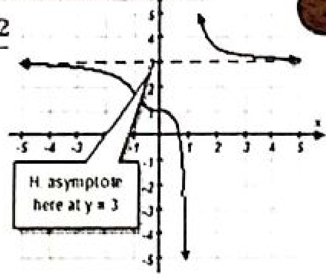
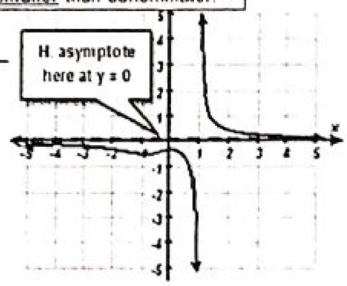
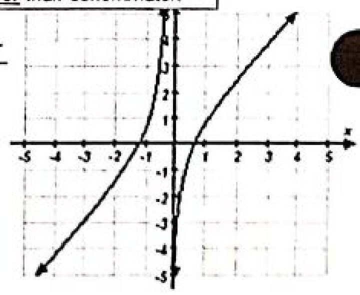
DOMAIN: $x \neq 1$ Range: $y \neq 4$

2. $f(x) = \frac{x^2+3x-4}{x^2+2x-8}$ $\frac{(x+4)(x-1)}{(x+4)(x-2)}$

VA: $x=2$
HA: $y=1$
HOLE: $(-4, \frac{5}{6})$



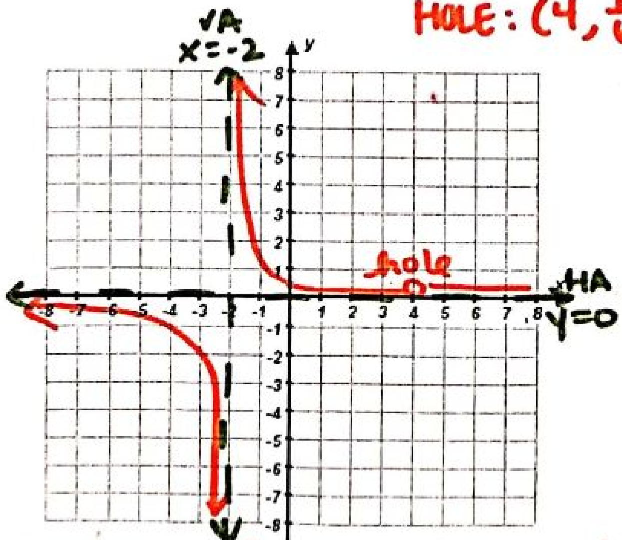
Domain: $x \neq 2, -4$ Range: $y \neq \frac{5}{6}, 1$

Potential Horizontal Asymptotes	Description	Example
Case #1:	A rational function that has a numerator polynomial with the **same degree** as the polynomial in the denominator creates a horizontal asymptote that passes through the y-axis at the quotient of the leading coefficients.	The degree of the numerator & denominator are the same **Graph:** $f(x) = \frac{6x^3 + x^2 - 2}{2x^3 - 2}$ **Analyze:** $f(x) = \frac{6x^3 + x^2 - 2}{2x^3 - 2}$ **Asymptote:** $y = \frac{6}{2}$ or $y = 3$ 
Case #2:	A rational function that has a polynomial in the **numerator that has a smaller degree** than the degree of the polynomial in the denominator creates a horizontal asymptote at y =0.	The degree of the numerator is smaller than denominator. **Graph:** $f(x) = \frac{2x^2 + 1}{3x^3 - 3}$ **Asymptote:** $y = 0$ 
Case #3:	A rational function that has a polynomial in the **numerator that has a larger degree** than the degree of the polynomial in the denominator does not have a horizontal asymptote.	The degree of the numerator is larger than denominator. **Graph:** $f(x) = \frac{5x^2 + 3x - 4}{4x^1 + 1}$ **Asymptote:** No Horizontal Asymptote 

Sketch a graph of the following rational functions. Label any vertical asymptotes, horizontal asymptotes, or holes. Use your calculator for additional assistance.

3. $f(x) = \frac{x-4}{x^2-2x-8}$ $\quad \frac{x-4}{(x-4)(x+2)}$

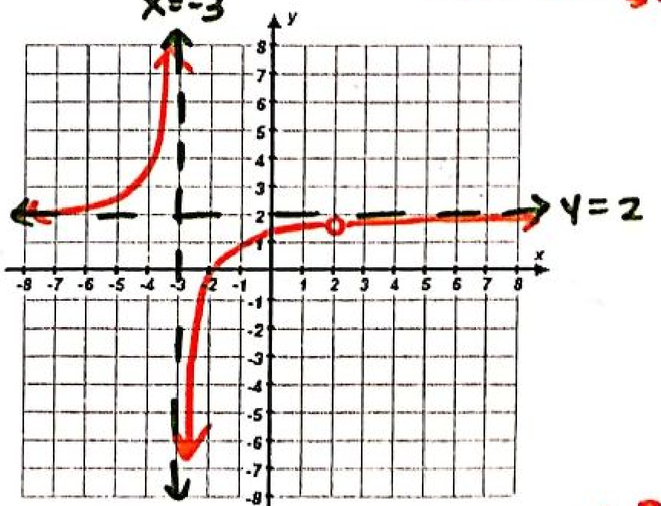
HA: y=0
VA: x=-2
HOLE: $(4, \frac{1}{6})$



Domain: $x \neq -2, 4$ Range: $y \neq 0, \frac{1}{6}$

4. $f(x) = \frac{2x^2-8}{x^2+x-6}$ $\quad \frac{2(x+2)(x-2)}{(x+3)(x-2)}$

VA: x=-3
HA: y=2
HOLE: $(2, \frac{8}{5})$



Domain: $x \neq -3, 2$ Range: $y \neq \frac{8}{5}, 2$