

4.1 Quadratic Regressions

Explore: Graphs Unmasked - Data Detective

Use the data sets below to construct **quadratic functions** that model the relationships between the variables. Label the equations for each data set and describe any key characteristics you observe from the graph. Remember to consider the vertex, axis of symmetry, maximum/minimum points, and y-intercept when analyzing the quadratic functions.

Using Graphing Calculators

Data Set 1: Projectile Motion	
Time (seconds) <i>L1</i>	Height (meters) <i>L2</i>
0	5
1	8
2	9
3	8
4	5

Steps:

Enter Data into L1 and L2 [stat, 1:edit]

Press [stat, >, 5: QuadReg]

To be able to graph the quadratic, enter Y1 [alpha, trace, enter] next to StoreReg

Quadratic Function Model: $y = -x^2 + 4x + 5$

Vertex: $(2, 9)$ **Maximum** or Minimum? Axis of Symmetry: $x = 2$ y-intercept: $(0, 5)$

How might the characteristics above relate to the actual situation of projectile motion?

The vertex represents the maximum height of the projectile. The y-intercept represents the launch height. The ^{positive} x-intercept represents the time it takes the projectile to hit the ground.

Using Desmos Graphing Calculator

Data Set 2: Population Growth	
Year	Population (thousands)
<i>x=0</i> 2000	250
<i>x=2</i> 2002	320
<i>x=4</i> 2004	420
<i>x=6</i> 2006	500
<i>x=8</i> 2008	550

Steps:

Click on the plus sign to add a table

Enter the data into the table under x1 and y1

In the next row type in $y_1 \sim ax_1^2 + bx_1 + c$ for the quadratic regression.

Note: you can use any form of the quadratic to do this but standard form works best. Also, when dealing with years it helps to use $x=0$ for the first year so your numbers won't be large.

Quadratic Function Model: $y = -1.07x^2 + 45.6x + 243$

Vertex $(22.2, 771)$ **Maximum** or Minimum? Axis of Symmetry: $x = 22.2$ y-intercept: $(0, 243)$

How might the characteristics above relate to the actual situation of population growth?

The y-intercept represents the estimated population in 2000 (the starting year). The vertex represents the ^{# of} years after 2000 the peak population occurs according to this model.

Complete the rest of the examples below using either method of quadratic regression:

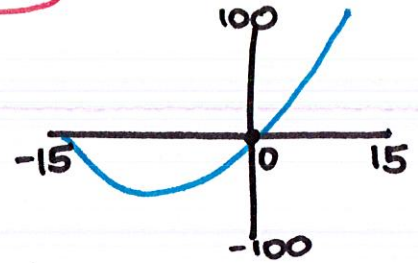
Data Set 3: Profit Analysis	
Production (units)	Profit (dollars)
0	0
1	10
2	25
3	40
4	55

Quadratic Function Model: $y = 0.714x^2 + 11.1x - 0.571$

Vertex: $(-7.8, -44.9)$ Maximum or minimum?

Axis of symmetry: $x = -7.8$

y-intercept: $(0, -0.571)$



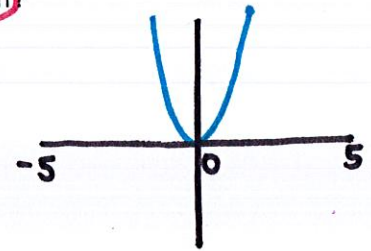
Data Set 4: Freefall Acceleration	
Time (seconds)	Distance (meters)
0	0
1	5
2	20
3	45
4	80

Quadratic Function Model: $y = 5x^2$

Vertex: $(0, 0)$ Maximum or minimum?

Axis of symmetry: $x = 0$

y-intercept: $(0, 0)$



Data Set 5: Sales Revenue	
Month	Revenue (thousands)
Jan	100
Feb	120
Mar	150
Apr	180
May	200

Quadratic Function Model: $y = 26x + 72$!

Vertex: ~~NOT QUADRATIC~~ Maximum or minimum?

Axis of symmetry: ~~NOT QUADRATIC~~

y-intercept: $(0, 72)$

Data Set 6: Temperature Change	
Time (hours)	Temperature (degrees Celsius)
0	20
1	18
2	15
3	12
4	10

Quadratic Function Model: $y = -2.6x + 20.2$!

Vertex: ~~NOT QUADRATIC~~ Maximum or minimum?

Axis of symmetry: ~~NOT QUADRATIC~~

y-intercept: $(0, 20.2)$

Are there any function models that surprised you? What do you notice about the rates of change in the tables and in the functions themselves?

Data Sets 5 & 6 have non-quadratic models because the rates of change are almost constant whereas for Data Sets 3 & 4 the increase in y gets larger for every unit increase in x . Data Set 4 increased much faster than Data Set 3.