3.4 Completing the Square Notes

Some quadratic equations in the form of $ax^2 + bx + c = 0$ can be solved easily by factoring. For example, the equation $x^2 + 6x - 16 = 0$ can be factored easily to $(x+8)(x-2) = 0$ to give solutions of $x = -8$ *and* $x = 2$.

When a quadratic equation cannot be factored using integers, you have two options. You can use the quadratic formula of you can use a method called **completing the square**. When a = 1 and b is an even number, completing the square is the way to go (when a >1, use the quadratic formula).

Example 1: Solve $x^2 + 8x - 10 = 0$ by completing the square.

Since it cannot be factored using integers, Write the equation in the form $ax^2 + bx = -c$	$x^2 + 8x - 10 = 0$ (undo) move the constant to right side $x^2 + 8x = 10$
Find $\frac{1}{2}$ of b and add the square of that number $(\frac{b}{2})^2$ to both sides of the equation Completing the Square Step	Think $b = 8$ $\frac{1}{2}b = 4$ and $4^2 = 16$ $x^2 + 8x = 10$ $x^2 + 8x + 16 = 10 + 16$
The left side is now a perfect square trinomial (PST), so factor it.	$(x+4)(x+4) = 26$ $(x+4)^2 = 26$
Find the square root of each side.	$(x+4)^2 = 26$ $x + 4 = \pm\sqrt{26}$
Solve for x	$x = -4 \pm \sqrt{26}$
Use a calculator to approximate the solutions, if necessary	$x \approx -4 \pm 5.099$ $x \approx 1.099$ *or* -9.099

Important Note: It is important that if a question asks you to complete the square. You are finding half of the "b" and squaring it. This is the number that will complete the square.

What number will complete the square? $x^2 + 12x + n$ $\frac{12}{2} = 6$ $6^2 = \boxed{36}$

① $x^2+10x+29=0$
$-29 \quad -29$
$x^2+10x+\underline{25} = -29+\underline{25}$
$\frac{10}{2}=5$ $5^2=25$
$(x+5)^2 = -4$
$\sqrt{(x+5)^2} = \pm\sqrt{-4}$
$x+5 = \pm 2i$
$-5 \quad -5$
$x = -5 \pm 2i$
$x=-5+2i$
$x=-5-2i$

② $x^2+8x-14=0$
$+4 \quad +4$
$x^2+8x+\underline{16} = 4+\underline{16}$
$\frac{8}{2}=4$ $4^2=16$
$(x+4)^2 = 20$
$\sqrt{(x+4)^2} = \pm\sqrt{20}$
$x+4 = \pm 2\sqrt{5}$
$-4 \quad -4$
$x = -4 \pm 2\sqrt{5}$
$x=-4+2\sqrt{5}$
$x=-4-2\sqrt{5}$
20 = 4 × 5, $\sqrt{4}=2$

③ $x^2-2x-33=0$
$+33 \quad +33$
$x^2-2x+\underline{1} = 33+\underline{1}$
$\frac{-2}{2}=-1$ $(-1)^2=1$
$\sqrt{(x-1)^2} = \pm\sqrt{34}$
$x-1 = \pm\sqrt{34}$
$+1 \quad +1$
$x = 1 \pm \sqrt{34}$
$x=1+\sqrt{34}$
$x=1-\sqrt{34}$

④ special ***
$\frac{4x^2}{4}+\frac{24x}{4}+\frac{37}{4}=\frac{0}{4}$ because $a \neq 1$ divide by a 1st
$x^2+6x+\frac{37}{4}=0$
$-\frac{37}{4} \quad -\frac{37}{4}$
$x^2+6x+\underline{9} = -\frac{37}{4}+\underline{9}$
$\frac{6}{2}=3$ $3^2=9$
$(x+3)^2 = -\frac{1}{4}$
$\sqrt{(x+3)^2} = \pm\sqrt{-\frac{1}{4}}$
$x+3 = \pm\frac{1}{2}i$
$-3 \quad -3$
$x = -3 \pm \frac{1}{2}i$
$x=-3+\frac{1}{2}i$
$x=-3-\frac{1}{2}i$