

2.9 Inverse Functions

inverse of a function: undoes the action(s) of another function.

example of inverse functions:

$$y = 2x + 3 \quad \text{and} \quad y = \frac{x-3}{2}$$

Fun Facts about inverses

- ① Suppose $f(x)$ & $g(x)$ are inverse functions
 $f(g(x)) = g(f(x)) = x$
- ② $f(x)$ and $g(x)$ are reflections of each other in the line $y=x$.
- ③ Notation f^{-1} or $f^{-1}(x)$
which means "f inverse" or "the inverse of f"

Steps to Finding an Inverse Function

Step 1: change $f(x) =$ to $y =$

Step 2: swap x & y x becomes y
 y becomes x

Step 3: Solve for y

Step 4: change $y =$ to $f^{-1}(x) =$

Ex.1 Find the inverse of $f(x) = -3x - 1$

$$y = -3x - 1$$

$$x = -3y - 1$$

$$+1 \qquad +1$$

$$\frac{x+1}{-3} = \frac{-3y}{-3}$$

$$\frac{x+1}{-3} = y \quad \rightarrow \quad f^{-1}(x) = \frac{x+1}{-3}$$

Y1 Find the inverse of $f(x) = 5 - 2x$

$$y = 5 - 2x$$

$$x = 5 - 2y$$

$$\frac{x-5}{-2} = \frac{-2y}{-2}$$

$$\frac{x-5}{-2} = y \quad \rightarrow \quad f^{-1}(x) = \frac{x-5}{-2}$$

Ex.2 Find the inverse of $g(x) = -3 + x^3$

$$y = -3 + x^3$$

$$x = -3 + y^3$$

$$+3 \quad +3$$

$$x+3 = y^3$$

$$\sqrt[3]{x+3} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+3} = y \quad \rightarrow \quad g^{-1}(x) = \sqrt[3]{x+3}$$

EX.3 Find the inverse of $f(x) = \frac{1}{x-2} - 1$

$$y = \frac{1}{x-2} - 1$$

$$x = \frac{1}{y-2} - 1$$

$$+1 \qquad +1$$

$$x+1 = \frac{1}{y-2}$$

$$y-2 = \frac{1}{x+1}$$

$$+2 \qquad +2$$

$$y = \frac{1}{x+1} + 2$$

$$f^{-1}(x) = \frac{1}{x+1} + 2$$

Note:
 $a = \frac{b}{c}$
is the same as $c = \frac{b}{a}$

YI Find the inverse of $g(x) = \frac{-1}{x+3} - 5$