

2.8 Operations with Functions

Suppose $f(x) = 2x + 3$ and $g(x) = -4x^2$

and $h(x) = \frac{6}{x}$

Add

write the sum & combine like terms

① $f + g$ $2x + 3 + -4x^2$
 $f(x) + g(x)$

Subtract

write the difference

distribute the minus

combine like terms

② $g - f$ $(-4x^2) - (2x + 3)$
 $g(x) - f(x)$ $-4x^2 - 2x - 3$

③ $h - f$ $(\frac{6}{x}) - (2x + 3)$
 $h(x) - f(x)$ $\frac{6}{x} - 2x - 3$

Multiply

④ gf $(-4x^2)(2x + 3)$
 $g \cdot f$ $-8x^3 - 12x^2$
 $g(x) \cdot f(x)$

Divide

⑤ $g \div f$ $\frac{-4x^2}{2x + 3}$
 $\frac{g}{f}$ or $\frac{g(x)}{f(x)}$

Suppose $f(x) = x^2 + 1$ $g(x) = -x + 5$

and $h(x) = \sqrt{x}$

Ex. 1

Find $f(2) \cdot g(2)$

$$\begin{array}{c} 5 \cdot 3 \\ \boxed{15} \end{array}$$

$$\begin{aligned} f(2) &= 2^2 + 1 \\ f(2) &= 5 \end{aligned}$$

$$\begin{aligned} g(2) &= -2 + 5 \\ g(2) &= 3 \end{aligned}$$

Y.T. 1

Find $h(4) + g(4)$

$$\begin{array}{c} 2 + 1 \\ \boxed{3} \end{array}$$

$$\begin{aligned} h(4) &= \sqrt{4} \\ h(4) &= 2 \end{aligned}$$

$$\begin{aligned} g(4) &= -4 + 5 \\ &= 1 \end{aligned}$$

Composition of Functions

evaluate $g(\underbrace{x+3}_{\text{input}}) = -(\overbrace{x+3}) + 5$

$$\begin{aligned} &= -x - 3 + 5 \\ &= -x + 2 \end{aligned}$$

Find $g(f(x))$

$$\begin{aligned} g(x^2+1) &= -(x^2+1) + 5 \\ &= -x^2 - 1 + 5 \\ &= -x^2 + 4 \end{aligned}$$

Notations for Compositions of Functions

$$\begin{array}{cc} g(f(x)) & f(g(x)) \\ \text{or} & \text{or} \\ (g \circ f)(x) & (f \circ g)(x) \end{array}$$

"g of f of x"

"f of g of x"

90% of the time $g(f(x)) \neq f(g(x))$
so order matters!

suppose $f(x) = x^2$ and $g(x) = 2x+3$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$

a) $(f \circ g)(x)$

$$\begin{aligned} f(g(x)) \\ f(2x+3) &= (2x+3)^2 \\ &= (2x+3)(2x+3) & (f \circ g)(x) &= 4x^2 + 12x + 9 \\ &= 4x^2 + 6x + 6x + 9 \\ &= 4x^2 + 12x + 9 \end{aligned}$$

b) $(g \circ f)(x)$

$$\begin{aligned} g(f(x)) & & (g \circ f)(x) &= 2x^2 + 3 \\ g(x^2) &= 2x^2 + 3 \end{aligned}$$

Ex. suppose $f(x) = -x + 5$ and $g(x) = 3 - 2x$

Find $f(g(x))$ and $g(f(x))$

a) $f(g(x))$ Replace inside function
 $f(3-2x)$ with what it is.

$$f(x) = -x + 5$$

$$f(3-2x) = -(3-2x) + 5$$

$$= -3 + 2x + 5$$

$$= 2x + 2$$

$$f(g(x)) = 2x + 2$$

b) $g(f(x))$

$$g(-x+5)$$

$$g(x) = 3 - 2x$$

$$g(-x+5) = 3 - 2(-x+5)$$

$$= 3 + 2x - 10$$

$$= 2x - 7$$

$$g(f(x)) = 2x - 7$$

Ex. Suppose $f(x) = 2x + 1$ and $g(x) = x^2$

Find $f(g(3))$ and $g(f(-1))$

a) $f(g(3))$
Find $g(3)$ first

$$g(3) = 3^2$$
$$g(3) = 9$$

take that output &
plug into outside function f

$$f(9) = 2(9) + 1$$
$$= 19$$

$$f(g(3)) = 19$$

c) find $g(g(2))$

$$g(2) = 2^2$$
$$g(2) = 4$$

$$g(4) = 4^2$$
$$g(4) = 16$$

$$g(g(2)) = 16$$

b) $g(f(-1))$
Find $f(-1)$ first

$$f(-1) = 2(-1) + 1$$
$$f(-1) = -1$$

$$g(-1) = (-1)^2$$
$$= 1$$

$$g(f(-1)) = 1$$

d) $g(f(5))$

$$f(5) = 2(5) + 1$$
$$= 11$$

$$g(11) = 11^2$$
$$= 121$$

$$g(f(5)) = 121$$