Name: ____________________ Date: __________ Period: ________

2.5 ________ **Exponential and Logarithmic Equations Continued**

Apply: A Caffeine Problem

The half-life of caffeine is approximately 5 hours; this means that approximately ½ of the caffeine in the bloodstream is eliminated every 5 hours. Suppose you drink a can of Instant Energy, a 16-ounce energy drink that contains 160 mg of caffeine. Suppose the caffeine in your bloodstream peaks at 160 mg.

1) How much caffeine will remain in your bloodstream after 5 hours? 10 hours? 1 hour? 2 hours? Record your answers in the table. Explain how you came up with your answers. (You can return to your answers later to make any corrections if you find your strategy was incorrect.)

Answers will vary.

Time (hours) since peak level reached	0	1	2	5	10
Caffeine in bloodstream (mg)	160	139	121	80	40

2) Write an exponential function f to model the amount of caffeine remaining in the blood stream t hours after the peak level. What does your exponent need to represent? How can you determine this exponent if you know the number of hours that has passed?

$f(t) = 160\left(\frac{1}{2}\right)^{\frac{t}{5}}$

Exponent needs to represent how many 5-hr periods have passed, so you must divide # of hours by 5.

3) Use the function you wrote in question (2) to check your answers for the table in question (1). Make any necessary corrections.

Answers will vary.

4) Determine the amount of caffeine remaining in the bloodstream 3 hours after the peak level. What about 8 hours after peak level? 20 hours?

t	3	8	20
$f(t)$	106	53	10

5) The actual half-life of caffeine varies among individuals and can be affected by other medications, BMI, and age. For example, some medications extend the half-life of caffeine to 8 hours. This means that ½ of the caffeine is eliminated from the bloodstream every 8 hours. Write a function for this new half-life time (assuming a peak level of 160 mg of caffeine).

$g(t) = 160\left(\frac{1}{2}\right)^{\frac{t}{8}}$

6) Determine the amount of caffeine in the bloodstream after 1 hour, 2 hours, 5 hours, 10 hours, and 20 hours. (Be sure to consider how many 8-hour time intervals are used in each time value.)

t	1	2	5	10	20
$g(t)$	147	135	104	67	28

7) Which half-life time results in the caffeine being eliminated faster? Explain why this makes sense.

The 5-hr half life time eliminates caffeine faster since 5 hours is a shorter time interval than 8 hours.

8) Consider again the function in question (2) resulting from a half-life of caffeine of 5 hours. Use the properties of exponents to rewrite the function so the exponent is just t, not $\frac{t}{5}$. Can you now determine the percent of caffeine that remains in the bloodstream each hour? Explain how.

$f(t) = 160\left(\frac{1}{2}\right)^{\frac{t}{5}} \Rightarrow f(t) = 160\left(\frac{1}{2}\right)^{\frac{1}{5}\cdot t} \Rightarrow f(t) = 160\left[\left(\frac{1}{2}\right)^{\frac{1}{5}}\right]^{t}$

Power of a Power Property

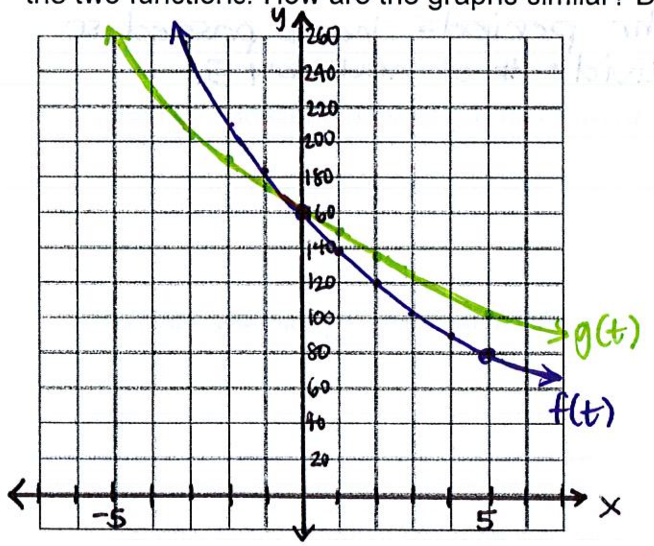
$\left(\frac{1}{2}\right)^{\frac{1}{5}} \approx 0.87055\ldots \Rightarrow f(t) = 160\,(0.87055\ldots)^{t}$

9) The function in question (5) referred to a half-life of 8 hours. Use properties of exponents to help you determine the percent of caffeine that remains in the bloodstream each hour?

$g(t) = 160\left(\frac{1}{2}\right)^{\frac{t}{8}} \Rightarrow g(t) = 160\left[\left(\frac{1}{2}\right)^{\frac{1}{8}}\right]^{t}$

$g(t) = 160\,(0.91700\ldots)^{t}$

10) Graph the functions from questions (2) and (5) on the same coordinate plane. Compare the graphs of the two functions. How are the graphs similar? Different?



Both graphs are exponential decay.

Both graphs have an asymptote of y=0.

11) Do the graphs intersect? Where?

The graphs intersect at (0,160).

In a previous lesson, we used the method of one-to-one property to solve exponential and logarithmic equations because we could rewrite the equation with the same base on both sides for the powers or logarithms. But sometimes this is not possible, so we use the inverse property to solve equations.

Inverse Properties:

$$a^{\log_a M} = M \qquad \qquad \log_a a^P = P$$

Don't forget:

Definition of a Logarithm: $y = \log_a x \;\; if\ and\ only\ if \;\; x = a^y \;\; for\ a > 0$

For the following equations, solve and give the exact solution and then give the answer correct to 3 significant figures.

Example 12: Solve. $3(2^x) = 42$	**Steps to Solving**	**Example 13:** Solve for x. $4e^{2x} - 3 = 2$
$\frac{3(2^x)}{3} = \frac{42}{3}$ $2^x = 14$	Isolate the power (base and exponent)	$+3 \quad +3$ $\frac{4e^{2x}}{4} = \frac{5}{4}$ $e^{2x} = 1.25$
$\boxed{x = \log_2 14}$ exact $\boxed{x \approx 3.81}$ approx.	Change to logarithmic form using the definition or take the logarithm of both sides with the same base as the power.	$\frac{2x}{2} = \frac{\ln 1.25}{2}$
	Finish solving for x, if necessary.	$\boxed{x = \frac{1}{2}\ln 1.25}$ exact
	For approximate answers, enter entire expression into calculator.	$\boxed{x \approx 0.112}$ approx.
Example 14: Solve $\log_5(3x+1) = 2$		**Example 15:** Solve: $\log_2 x - \log_2 3 = 5$
$3x + 1 = 5^2$	Condense all of the logarithmic terms to a single logarithm using properties of logarithms.	$\log_2\left(\frac{x}{3}\right) = 5$ quotient property
$3x+1 = 25$ $-1 \quad -1$ $\frac{3x}{3} = \frac{24}{3}$	Change to exponential form using the definition or 'exponentiate' both sides.	$\frac{x}{3} = 2^5$
$\boxed{x = 8}$	Finish solving for x, if necessary.	$\frac{x}{3} = 32$ $3 \cdot \frac{x}{3} = 32 \cdot 3$ $\boxed{x = 96}$