Name: ______________________________________ Date: _______________ Period: ________

________ **Exponential & Logarithmic Equations**

Activator: One-to-One Property Exponential Equations

Some exponential equations can be solved using the one-to-one property stated below:

$$\textbf{If } b^m = b^n, \textbf{ then } m = n$$

Use this property and the properties of exponents to solve the exponential equations in the following examples.

Let a and b be real numbers and let m and n be integers.

PRODUCT OF POWERS PROPERTY	$a^m \cdot a^n = a^{m+n}$
POWER OF A POWER PROPERTY	$(a^m)^n = a^{mn}$
POWER OF A PRODUCT PROPERTY	$(ab)^m = a^m b^m$
NEGATIVE EXPONENT PROPERTY	$a^{-m} = \frac{1}{a^m}, a \neq 0$
ZERO EXPONENT PROPERTY	$a^0 = 1, a \neq 0$
QUOTIENT OF POWERS PROPERTY	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
POWER OF A QUOTIENT PROPERTY	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

1. $5^{3x} = 5^{7x-2}$

2. $9^{-3x} = 3^6$

3. $\frac{1}{8} = 4^{3x+1}$

4. $9^{y+2} = 27^{4y-2}$

5. $e^{2x} = e^{3x} \cdot \frac{1}{e^2}$

Engage: Properties of Logarithms

Just like there are properties of exponents, there are also properties of logarithms. Some of them stem directly from a specific exponent property.

The Basic Properties

Related Exponent Property	Name of Logarithm Property	Property	Example 1	Example 2
	Argument of b	$\log_b b = 1$	$\log_{14} 14 =$	$\ln e =$
	Argument of 1	$\log_b 1 = 0$	$\log 1 =$	$\log_{32} 1 =$

The Main Properties: Note these properties are used to 'expand' and 'condense' logarithmic expressions.

Related Exponent Property	Name of Logarithm Property	Property	Example 1	Example 2
Product of Powers	**Product Property**	$\log_b(u \cdot v) = \log_b u + \log_b v$	**Condense:** $\log_5 6 + \log_5 7 =$	**Expand:** $\log_2 63 =$
Quotient of Powers	**Quotient Property**	$\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$	**Condense:** $\log_4 84 - \log_4 12 =$	**Expand:** $\log 9 =$
Power of a Power	**Power Property**	$\log_b u^k = k \log_b u$	**Condense:** $2\log_3 8 =$	**Expand:** $\log_5 6^x =$

Let's look at some more examples:

Expanding Logarithmic Expressions

Expand each expression as a sum and/or difference of logarithms. Express powers as factors.

6) $\log_a(u^2v^3)\,, u > 0, v > 0$	7) $\log_2\left(\frac{x^3}{x-3}\right)\ \ , x > 3$

Condensing Logarithmic Expressions

Condense each expression into a single logarithm.

8) $3\log_5 u + 4\log_5 v$	9) $2\log_b(5x^3) - \frac{1}{2}\log_b(2x+3)$

<table>
<tr>
<td><u>Order</u>
↓ Quotient Rule
Product Rule
Power Rule ↑</td>
<td rowspan="2">DO NOT make these common errors/mistakes:
$\log_b(x+y) \neq \log_b x + \log_b y$
$\log_b(x-y) \neq \log_b x - \log_b y$
$(\log_b x)^r \neq r \log_b x$
$\log_b x - \log_b y \neq \frac{\log_b x}{\log_b y}$</td>
</tr>
<tr>
<td>All of these properties are very useful when we have to solve logarithmic equations. We can use these properties to simplify our equations and then use the definition of a logarithm to rewrite them in exponential form.</td>
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Some logarithmic equations can be solve using one-to-one properties of logarithms. If you can write both sides of the equation as a single logarithm with the same base then you can use the one-to-one property (for logarithms), that says,

If $\log_b m = \log_b p$, then $m = p$

Special Note: *Unlike exponential equations, you will have to check the solutions to logarithmic equations. The domain of logarithms is **not** all real numbers. Extraneous solutions make the argument of **<u>any</u>** logarithm (in the original equation) less than or equal to zero.*

10) $\log_2 2x = \log_2 100$	11) $\ln(x+4) = \ln 7$
12) $\log_7 3 + \log_7 x = \log_7 32$	13) $\log x - \log 6 = 2 \log 4$

When you want to solve exponential and logarithmic equations where both sides cannot be written as the same base (exponential equations) or there aren't logarithms on both sides of the equation (logarithmic equations), you will need to use inverse properties, we will talk about this in the next lesson.