Name: ______________________ Date: ____________ Period: __________

2.4 Investigating Exponential & Logarithmic Functions

Engage: Paper Folding

1) Take a large rectangular sheet of paper and fold it in half. You now have two equal-sized sections each with an area that is half the original area. Fold the paper in half again.

a. How many sections of paper do you have?

4

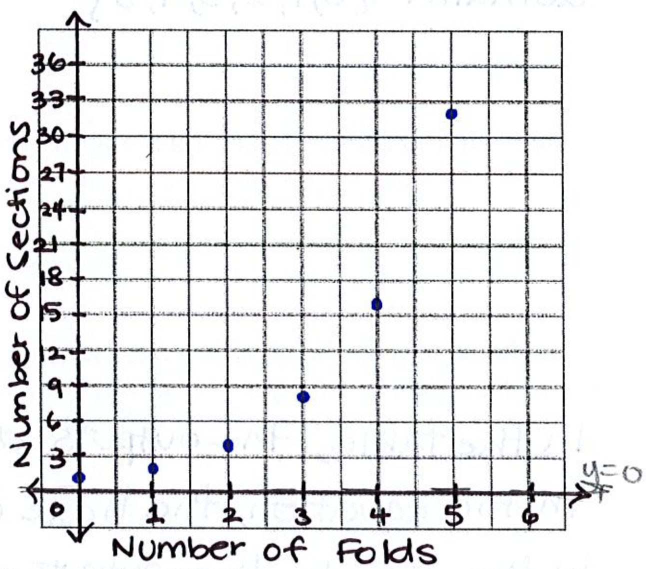
b. What is the area of each section compared to the area of the original piece of paper?

$\frac{1}{4}$

2) Continue this process until you cannot fold the paper anymore. Fill in the table below as you go.

Number of Folds	0	1	2	3	4	5
Number of Sections	1	2	4	8	16	32
Area of each section compared to original paper area	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

3) On graph paper let the horizontal axis represent the number of folds. Let the vertical axis represent the number of sections. Plot the points (# of folds, # of sections).



a. Does it make sense to connect these points with a smooth curve? Why or why not?

No, because the context here is number of folds which must be a positive integer (not a decimal).

b. Is the relationship between the number of folds and the number of sections a function? Why or why not?

Yes, every input has exactly one output.

c. What is the domain of this function?

domain: {0, 1, 2, 3, 4, 5}

d. Write the function f for the number of sections of paper you will have after x folds.

$f(x) = 2^x$

e. Use your function to determine the number of sections you would have if you were able to fold the paper 15 times.

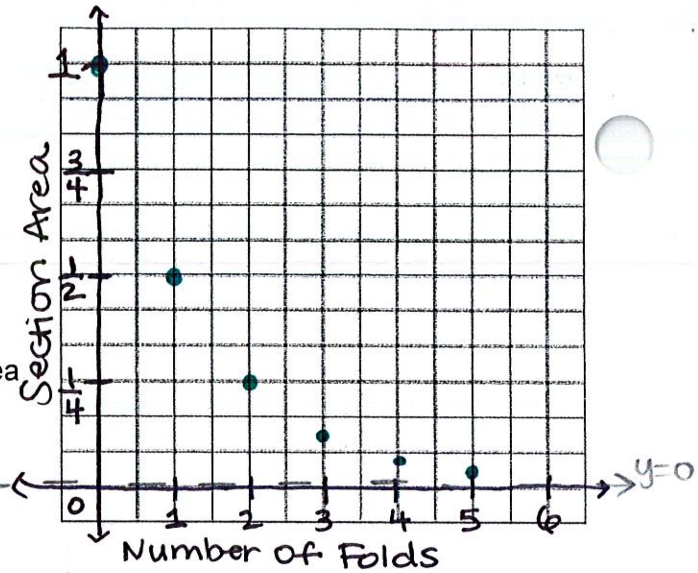
$f(15) = 2^{15}$

$f(15) = 32768$

f. The function f is an example of exponential growth. What do you notice about the table, equation, and graph of an exponential growth function?

In the table, the outputs are increasing (multiplying by 2).
In the equation, the base of the power is 2.
In the graph, the outputs are increasing rapidly, there is a steep curve.

4) Next, plot the points (# of folds, section area). Let the horizontal axis represent the number of folds; let the vertical axis represent the area of the section created.



a. Does it make sense to connect these points with a smooth curve? Why or why not?

No, because number folds must be a positive integer.

b. Is the relationship between the number of folds and the area of a section a function? Why or why not?

Yes, every input has exactly one output.

c. What is the domain of this function?

domain: $\{0,1,2,3,4,5\}$

d. Write the function g for the section area you will have after x folds.

$g(x) = 2^{-x}$ or $g(x) = \left(\frac{1}{2}\right)^{x}$

e. Use your function to determine the area of a section as compared to the area of the original paper if you were able to fold the paper 15 times.

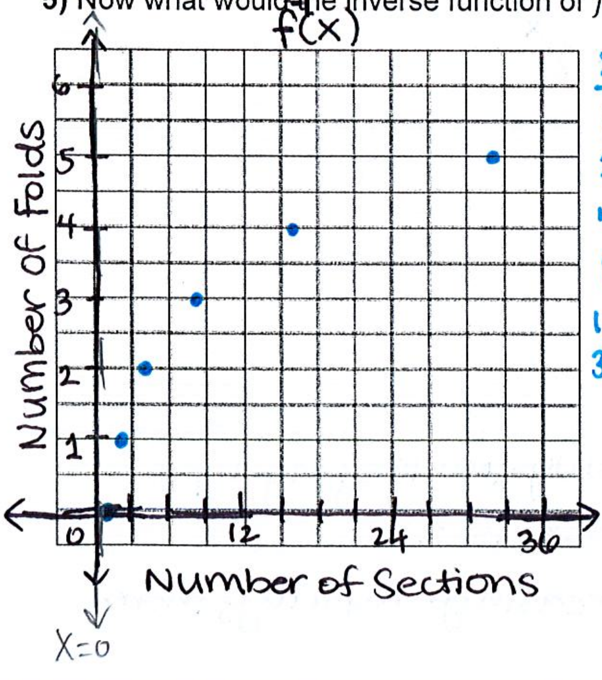
$g(15) = 2^{-15}$

$g(15) = \frac{1}{32768}$

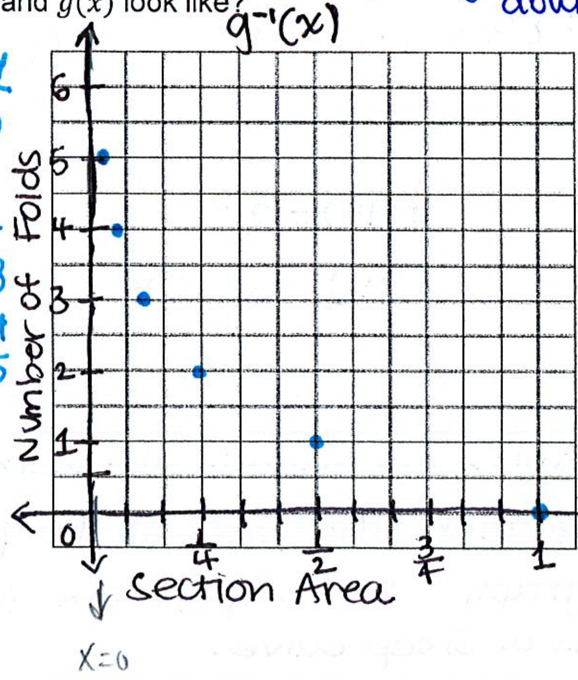
f. The function g for the area of a section is an example of exponential decay. What do you notice about the table, equation, and graph of an exponential decay function?

In the table, the outputs are dividing by 2 and decreasing.
In the equation, the base of the power is $\frac{1}{2}$ (it's a # between 0 & 1).
In the graph, the outputs are decreasing rapidly & curving downward.

5) Now what would the inverse function of $f(x)$ and $g(x)$ look like?



X	Y
1	0
2	1
4	2
8	3
16	4
32	5



X	Y
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2
$\frac{1}{8}$	3
$\frac{1}{16}$	4
$\frac{1}{32}$	5

How are these functions going to be defined? They are going to be defined using logarithms.

Logarithmic functions are inverses of exponential functions. In fact, the definition of a logarithm is tied to the definition of an exponential expression as such:

$$b^k = a, if\ and\ only\ if\ \log_b a = k$$

This definition can be used to help find inverses of exponential and logarithmic functions algebraically and eventually solve equations.

6) What would be the equations for inverse functions of $f(x)$ and $g(x)$?

original function	inverse function	how you say it.
$f(x) = 2^x$	$f^{-1}(x) = \log_2 x$	"log base 2 of x"
$g(x) = \left(\frac{1}{2}\right)^x$	$g^{-1}(x) = \log_{\frac{1}{2}} x$	"log base ½ of x"

Now, let's talk about and practice some basic logarithm concepts:

A logarithmic expression has **THREE PARTS**: 1) the word log 2) the base 3) the argument
The logarithm is not complete without all three parts.

$$\log_b a$$

Some things to consider:

- You cannot evaluate a logarithm that has a **ZERO** or **NEGATIVE** argument.
- Some logarithms are impossible to find without a calculator.

However, there are two logarithms where the base is **hidden** (not written explicitly but implied):

The common logarithm (base 10)
$\log 120$

The natural logarithm (base e)
$\ln 15$

These are also the logarithms that are on the face plate of the graphing calculator.

Write each equation in **exponential** form.		Write each equation in **logarithmic** form.	
a) $\log_3 9 = 2$ $3^2 = 9$	b) $\log_4\left(\frac{1}{16}\right) = -2$ $4^{-2} = \frac{1}{16}$	c) $8^{\frac{4}{3}} = 16$ $\log_8 16 = \frac{4}{3}$	d) $12^1 = 12$ $\log_{12} 12 = 1$
Evaluating Logarithms (No Calculator)			
e) $\log_2 64 = 6$ because $2^6 = 64$	f) $\log 100 = 2$ because $10^2 = 100$	g) $\log_{64} 4 = \frac{1}{3}$ because $64^{\frac{1}{3}} = 4$	h) $\log_9 \frac{1}{81} = -2$ because $9^{-2} = \frac{1}{81}$
Evaluating Logarithms (Calculator)			
i) $\log_2 54 \approx 5.75$	j) $\log 294 \approx 2.47$	k) $\log_4 136 \approx 3.54$	please note on an older version of the calculator or handheld calculator, you must use change of base formula to evaluate: $\log_b a = \frac{\ln a}{\ln b} = \frac{\log a}{\log b}$

To evaluate logs in calculator: Math ▲ A: logBASE(OR Shortcut: alpha window 5: logBASE(