Name: ______________________ Date: ____________ Period: __________

2.4 Investigating Exponential and Logarithmic Functions Continued

Explore: The Graphs

A basic [or parent] exponential function is $y = b^x$ *where* $b > 0$ *and* $b \neq 1$. An exponential function returns powers of the base number b. The input of the exponential function is the exponent, and the output is the number obtained when the base number is raised to that exponent.

1) Let's consider the four basic exponential functions below. Complete the tables and graph them on the coordinate axes.

$y=2^x$

x	y
-1	0.5
0	1
1	2
2	4

$y=3^x$

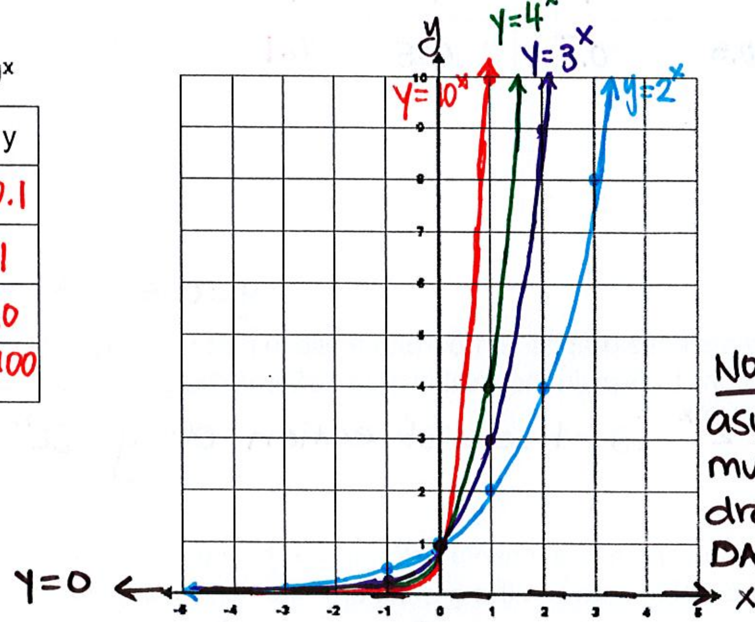
x	y
-1	$0.\bar{3}$
0	1
1	3
2	9

$y=4^x$

x	y
-1	0.25
0	1
1	4
2	16

$y=10^x$

x	y
-1	0.1
0	1
1	10
2	100



Note: asymptotes must be drawn with a DASHED Line.

2) What common characteristics of these functions do you notice? Determine the domain and range of the functions and any intercepts. Also describe any characteristics of their graphs such as increasing/decreasing, asymptotes, end-behavior, etc.

Function	Domain	Range	Intercepts	Other Characteristics		How does the graph of the exponential function change as the base b changes?
$y = 2^x$	$x \in \mathbb{R}$	$y > 0$	y-int (0,1)	increasing	asymptote @ y=0	The graph gets steeper as b increases.
$y = 3^x$	$x \in \mathbb{R}$	$y > 0$	↓	↓	↓	
$y = 4^x$	$x \in \mathbb{R}$	$y > 0$				
$y = 10^x$	$x \in \mathbb{R}$	$y > 0$				

NO x-int.

(closer to the y-axis).

3) The symbol e represents the irrational number 2.718281828…. Recall an irrational number is represented by a non-terminating, non-repeating decimal number. e is one of those important numbers in mathematics like π that keeps showing up in all kinds of places. $y=e^x$ is the natural exponential function. Use graphing technology to graph $y = 2^x$, $y = 3^x$, and $y = e^x$. How do their graphs compare? What do you notice about the graph of $y = e^x$ in relationship to the graphs of $y = 2^x$ and $y = 3^x$?

All graphs are similar but $y=e^x$ fits in between $y=2^x$ and $y=3^x$ since $2<e<3$. It is also important to note that $y=e^x$ is closer to $y=3^x$ than $y=2^x$ because e is closer to 3 than 2.

4) Let's consider the four basic exponential functions below. Complete the tables and graph them on the coordinate axes.

$y = 2^{-x}$

x	y
-2	4
-1	2
0	1
1	0.5

$y = 3^{-x}$

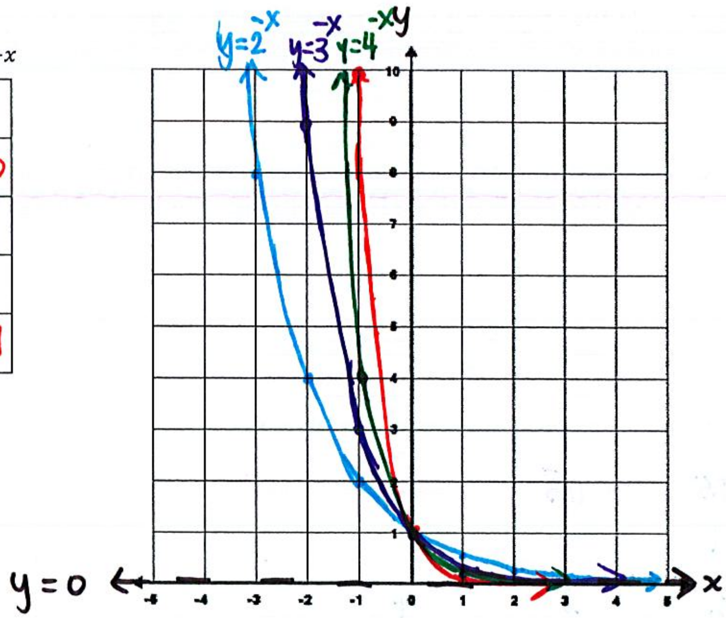
x	y
-2	9
-1	3
0	1
1	$0.\overline{3}$

$y = 4^{-x}$

x	y
-2	16
-1	4
0	1
1	0.25

$y = 10^{-x}$

x	y
-2	100
-1	10
0	1
1	0.1



5) How do these graphs compare to those in part (1) above? Use what you know about transformations of functions to explain the relationship between the graphs of $y = 2^x$ and $y = 2^{-x}$.

$y=2^{-x}$ is the reflection of $y=2^x$ in the y-axis.

6) Does the same relationship hold for $y = 3^x$ and $y = 3^{-x}$? For $y = 4^x$ and $y = 4^{-x}$? In general, what is the relationship between the graphs of $y = b^x$ and $y = b^{-x}$?

Yes, in general $y=b^{-x}$ is a reflection of $y=b^x$ over/in the y-axis.

7) Graph $y = \left(\frac{1}{2}\right)^x$ on the calculator. Compare its graph to $y = 2^{-x}$. What do you observe?

They are the same graph.

Use properties of exponents to explain the relationship between the two functions.

$y = 2^{-x} \Rightarrow y = \frac{1}{2^x} \Rightarrow y = \left(\frac{1}{2}\right)^x$

Negative exponents create reciprocals.

Do your observations about the graphs of $y = \left(\frac{1}{2}\right)^x$ and $y = 2^{-x}$ make sense?

Yes.

8) Graph $y = -2^x$. Compare its graph to $y = 2^x$. What do you observe? Use what you know about transformations of functions to explain the relationship between the graphs of $y = -2^x$ and $y = 2^x$.

$y=-2^x$ is a reflection of $y=2^x$ in/over the x-axis.

In general, $y=-b^x$ is the reflection of $y=b^x$ in the x-axis.

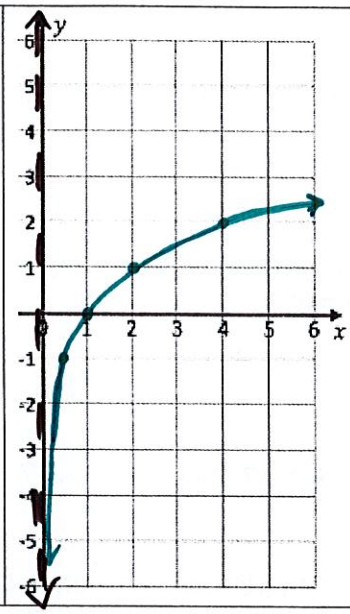
9) Remember that logarithmic functions are the inverse of exponential functions. Consider the basic logarithmic function, $y = \log_2 x$. Which function is it the inverse of?

$Y = 2^x$

10) Complete the table below and graph the function and describe its domain, range, intercepts and other characteristics.

$y = \log_2 x$

x	y
$\frac{1}{2}$	-1
1	0
2	1
4	2



X=0

Domain:

$X > 0$

Range:

$Y \in \mathbb{R}$

Intercepts:

x-int: (1, 0) No y-intercept

Other characteristics:

- increasing
- asymptote @ $x = 0$

11) Let's compare the graphs of a basic logarithmic function to other functions that adds a constant to the argument.

```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\Y1=log2(X)
■\Y2=log2(X+3)
■\Y3=log2(X+1)
■\Y4=log2(X-4)
■\Y5=log2(X-2)
■\Y6=
■\Y7=
■\Y8=
```

Since $g(x) = \log_2 x$ is the parent function.

Describe the effect of $g(x - k)$ when k is positive.

translates right k units

Describe the effect of $g(x - k)$ when k is negative.

translates left k units

12) Let's compare the graphs of a basic logarithmic function to other functions that adds a constant to the logarithm.

```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\Y1=log2(X)
■\Y2=log2(X)+2
■\Y3=log2(X)+4
■\Y4=log2(X)-1
■\Y5=log2(X)-3
■\Y6=
■\Y7=
■\Y8=■
```

Since $g(x) = \log_2 x$ is the parent function.

Describe the effect of $g(x) + k$ when k is positive.

translates up k units

Describe the effect of $g(x) + k$ when k is negative.

translates down k units