

## Investigating Exponential and Logarithmic Functions Continued

### Explore: The Graphs

A basic [or parent] exponential function is  $y = b^x$  where  $a > 0$  and  $a \neq 1$ . An exponential function returns powers of the base number  $b$ . The input of the exponential function is the exponent, and the output is the number obtained when the base number is raised to that exponent.

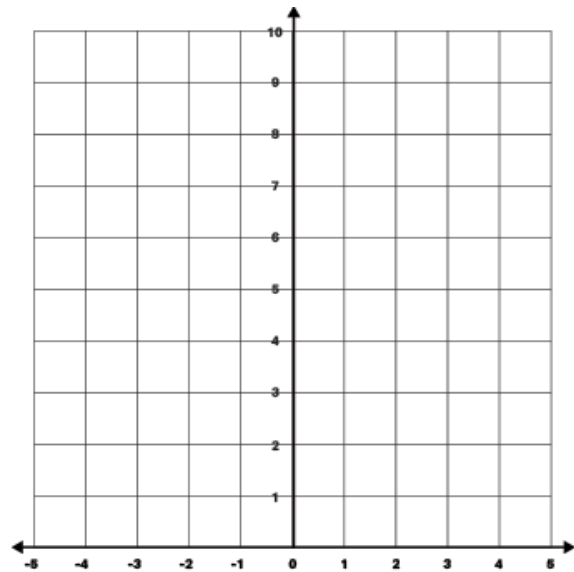
1) Let's consider the four basic exponential functions below. Complete the tables and graph them on the coordinate axes.

x	y
-1	
0	
1	
2	

x	y
-1	
0	
1	
2	

x	y
-1	
0	
1	
2	

x	y
-1	
0	
1	
2	



2) What common characteristics of these functions do you notice? Determine the domain and range of the functions and any intercepts. Also describe any characteristics of their graphs such as increasing/decreasing, asymptotes, end-behavior, etc.

Function	Domain	Range	Intercepts	Other Characteristics	How does the graph of the exponential function change as the base $b$ changes?
$y = 2^x$					
$y = 3^x$					
$y = 4^x$					
$y = 10^x$					

3) The symbol  $e$  represents the irrational number 2.718281828.... Recall an irrational number is represented by a non-terminating, non-repeating decimal number.  $e$  is one of those important numbers in mathematics like  $\pi$  that keeps showing up in all kinds of places.  $y=e^x$  is the natural exponential function. Use graphing technology to graph  $y = 2^x$ ,  $y = 3^x$ , and  $y = e^x$ . How do their graphs compare? What do you notice about the graph of  $y = e^x$  in relationship to the graphs of  $y = 2^x$  and  $y = 3^x$ ?

4) Let's consider the four basic exponential functions below. Complete the tables and graph them on the coordinate axes.

$$y = 2^{-x}$$

x	y
-2	
-1	
0	
1	

$$y = 3^{-x}$$

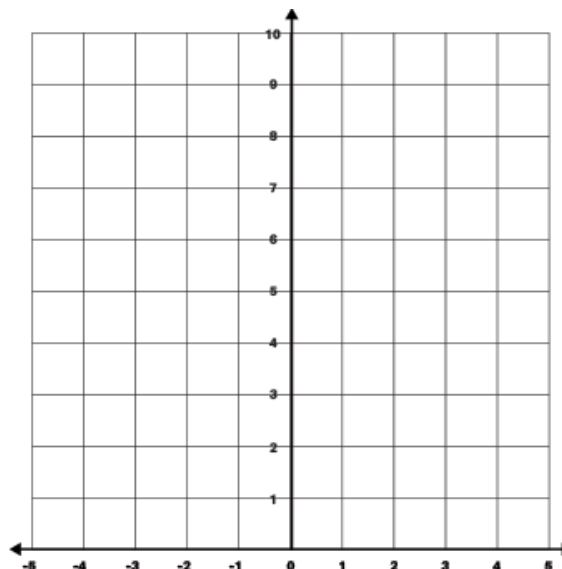
x	y
-2	
-1	
0	
1	

$$y = 4^{-x}$$

x	y
-2	
-1	
0	
1	

$$y = 10^{-x}$$

x	y
-2	
-1	
0	
1	



5) How do these graphs compare to those in part (1) above? Use what you know about transformations of functions to explain the relationship between the graphs of  $y = 2^x$  and  $y = 2^{-x}$ .

6) Does the same relationship hold for  $y = 3^x$  and  $y = 3^{-x}$ ? For  $y = 4^x$  and  $y = 4^{-x}$ ? In general, what is the relationship between the graphs of  $y = b^x$  and  $y = b^{-x}$ ?

7) Graph  $y = \left(\frac{1}{2}\right)^x$  on the calculator. Compare its graph to  $y = 2^{-x}$ . What do you observe?

Use properties of exponents to explain the relationship between the two functions.

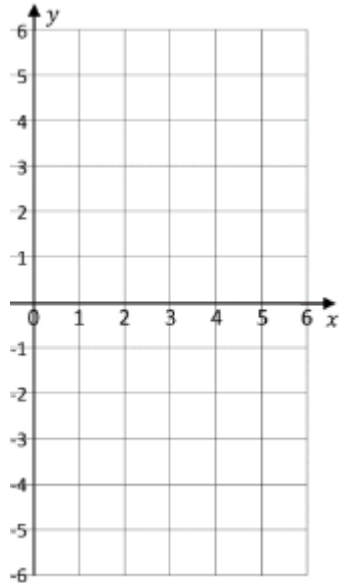
Do your observations about the graphs of  $y = \left(\frac{1}{2}\right)^x$  and  $y = 2^{-x}$  make sense?

8) Graph  $y = -2^x$ . Compare its graph to  $y = 2^x$ . What do you observe? Use what you know about transformations of functions to explain the relationship between the graphs of  $y = -2^x$  and  $y = 2^x$ .

9) Remember that logarithmic functions are the inverse of exponential functions. Consider the basic logarithmic function,  $y = \log_2 x$ . Which function is it the inverse of?

10) Complete the table below and graph the function and describe its domain, range, intercepts and other characteristics.

$y = \log_2 x$	
x	y
$\frac{1}{2}$	
1	
2	
4	



Domain:

Range:

Intercepts:

Other characteristics:

11) Let's compare the graphs of a basic logarithmic function to other functions that adds a constant to the argument.

<pre> NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=log2(X) Y2=log2(X+3) Y3=log2(X+1) Y4=log2(X-4) Y5=log2(X-2) Y6= Y7= Y8=         </pre>	<p>Since <math>g(x) = \log_2 x</math> is the parent function.</p> <p>Describe the effect of <math>g(x - k)</math> when <math>k</math> is positive.</p> <p>Describe the effect of <math>g(x - k)</math> when <math>k</math> is negative.</p>
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12) Let's compare the graphs of a basic logarithmic function to other functions that adds a constant to the logarithm.

<pre> NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=log2(X) Y2=log2(X)+2 Y3=log2(X)+4 Y4=log2(X)-1 Y5=log2(X)-3 Y6= Y7= Y8=         </pre>	<p>Since <math>g(x) = \log_2 x</math> is the parent function.</p> <p>Describe the effect of <math>g(x) + k</math> when <math>k</math> is positive.</p> <p>Describe the effect of <math>g(x) + k</math> when <math>k</math> is negative.</p>
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