Name: ____ Date: ___________ Period: _____

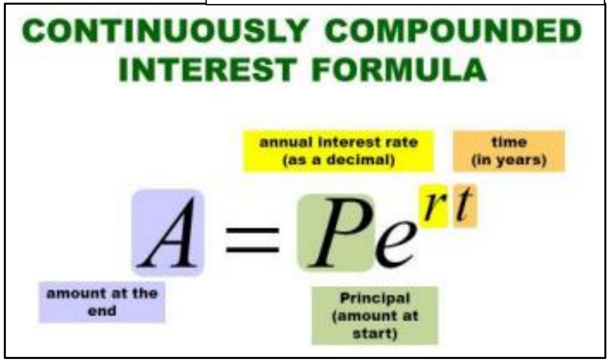
___________ **Introducing Exponential Functions**

Apply: Driving Question – How is the number *e* used in everyday life?

General Formula: $y = ae^{\pm kt}$
k > 0: _________________
k < 0: _________________

Understanding e in Real-Life contexts

The exponential function e in the compound interest formula is a mathematical constant approximately equal to 2.71828. It is often encountered in various scientific disciplines, including chemistry, physics, and biology. Chemical reactions and population growth in microbiology use exponential rate laws.



1) A student is studying the growth of a bacterial population in a petri dish. The initial population of bacteria is 100 cells, and the population doubles every 2 hours. The growth of the bacterial population follows an exponential growth model. The student wants to calculate the population size after 6 hours.

Assumptions:

The growth of the bacterial population follows exponential behavior.

The growth process is occurring at a constant temperature and under favorable conditions.

The volume or nutrient concentration in the petri dish remains constant throughout the growth process.

The growth constant (k) for the bacterial population is approximately 0.347.

a) Use the continuously compound interest formula, which is analogous to the exponential growth formula, to determine population size after 6 hours. b) Calculate the population size after 6 hours.

2) Radon is a radioactive gas used in cancer therapy to treat tumors. The radioactive decay of radon-222 can be modeled by the function:

$$A = Ce^{-0.1813t}$$

where *A* is the amount remaining, *C* is the original amount and *t* is the number of days.

If 10 mg of radon-222 remain after 5 days, how much was originally there?

Modeling Practice Problems

3) A sample of bacteria started with 4 million cells and each day the concentration doubles. Write a model for the concentration of a sample of bacteria (in millions per milliliter) after x days.

4) The growth of an investment is shown in the table below.

x (years)	0	5	10	15	20
y (dollars)	100	300	900	2700	8100

Suppose we want to build an exponential growth model for this investment in the form $f(x) = Ca^x$. What would the values of C and a be?

5) Esther purchased a used car, a Ford Focus, for $8400. The car is expected to decrease in value by 20% per year over the next 10 years. (a) What would be the decay factor? (b) How much would the car be worth 3 years after Esther purchased the car? (c) Give the domain and range for the model according to this situation.

6) Suppose Michael has $6000 to invest. Portfolio A offers a rate of 9% per annum compounded quarterly and Portfolio B offers a rate of 7.5% per annum compounded continuously. (a) Write the models for both portfolios. (b) Suppose Michael wants to invest his money in an account for 12 years, how much will each portfolio yield?

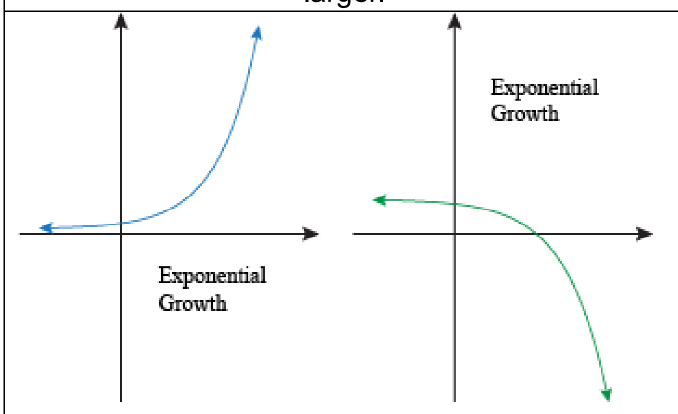
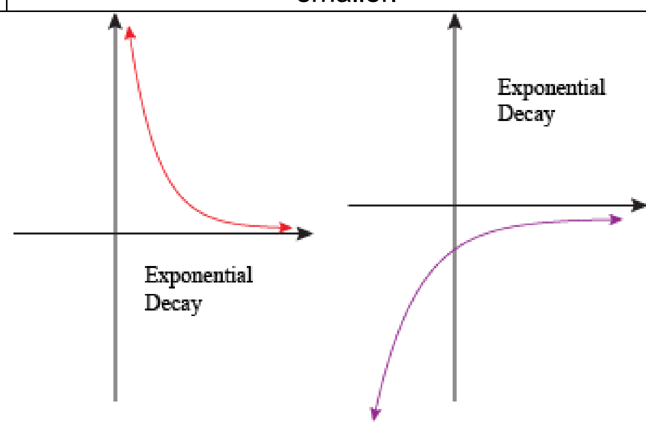
An **exponential function** is any function of the form:

$$y = ab^x$$

Where the exponent is a variable expression, $a \neq 0 \; and \; b > 0$.

a is the initial value **b** is the growth/decay factor **x** is the input & **y** the output

There are two types of exponential functions:

Exponential Growth	**Exponential Decay**
$b > 1$ In this situation, the magnitudes of the outputs get larger.	$0 < b < 1$ In this situation, the magnitude of the outputs get smaller.
	

Exponential Functions are frequently used in real-world problems like population growth or compound interest. This brings in another parameter to your equation that is usually important for the situation.

$\boldsymbol{b = 1 + r}$ for exponential growth	$\boldsymbol{b = 1 - r}$ for exponential decay
Where r represents your rate of increase or rate of decrease respectively. Sometimes r is called the **percent rate of change**. It is important to note that r is in decimal form in the expression and in itself is not positive or negative.	

Modeling with Exponential Functions

Below you will find typical exponential models and a description of the parameters. Remember in general there is always a 'multiplier' which will be raised to an exponent.

Name and Formula	Description of the Variables	Notes
Exponential Growth $y = ab^x$ $y = a(1+r)^t$	a – initial value b – growth factor (b>1) r – rate of increase t – time elapsed y – amount (final value) after time elapsed	
Exponential Decay $y = ab^x$ $y = a(1-r)^t$	a – initial value b – decay factor (0<b<1) r – rate of decrease t – time elapsed y – amount (final value) after time elapsed	
Half-Life $N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ $N(t) = N_0e^{kt}\ for\ k < 0$	N(t) – quantity remaining N_0 – initial quantity t – elapsed time h – half-life of the substance k – rate of decay	
Compound Interest $A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$	A – value of account after compounding P – original amount invested r – annual interest rate (as a decimal) n - # of compounding periods per year t – time elapsed (in years)	
Newton's Law of Cooling $u(t) = T + (u_0 - T)e^{kt}$ $for\ k < 0$	u – temperature of a heated object T – constant temperature of surrounding medium u_0 – initial temperature of the heated object k – negative constant	

Some Considerations to Think About When Building a Model		
When building your models, think of the appropriate domain that fits the situation. Does it make sense to include negative numbers? Does it make sense to include all real numbers as supposed to just positive integers?	Can the model be used indefinitely or are there reasonable restrictions for the range? Use tools when appropriate to help solve problems.	When answering questions based on your model, make sure your results make sense in the problem. Does your output need to be rounded down or up to the nearest integer to make sense?