Name: ______________________ Date: __________ Period: __________

2.2 Inverses of Functions Continued

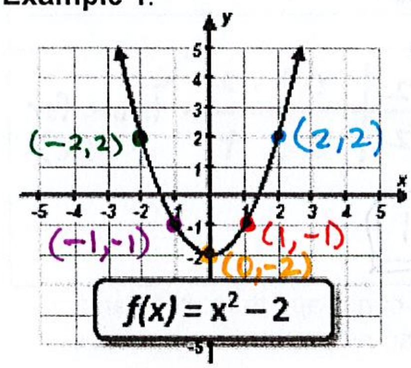
Inverse Relations

The inverse of a function swaps all the x and y coordinates of the original function to create a new relation. This relation will only be considered a function if it also passes the vertical line test meaning the original function was **one-to-one**. Then we will denote the new function, $f^{-1}(x)$ which we call "f-inverse of x" (assuming the original function was $f(x)$).

Finding Inverses Graphically

Using the equation and graph of the function, draw the graph of the inverse on the grid provided. Discuss whether the inverse is a function. If it not, think about how you could restrict the domain of the original function so that the inverse will be a function.

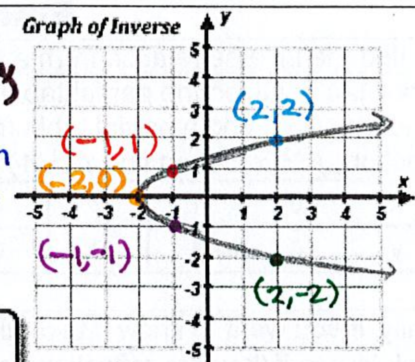
Example 1:



$(x,y) \longrightarrow (y,x)$

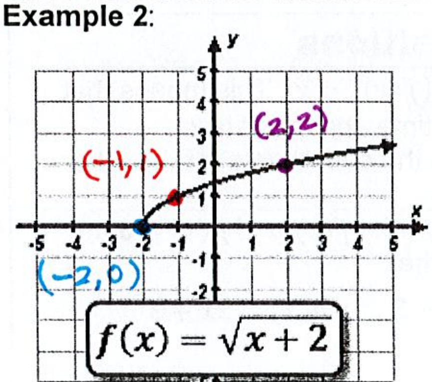
"SWAP THE COORDINATES"

If we restrict the domain of f(x) to either $x \leq 0$ or $x \geq 0$ then the inverse will be a function.

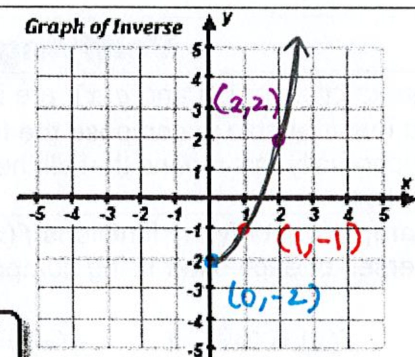


Is the Inverse a Function?
YES NO

Example 2:



Is the Inverse a Function?
YES NO



Finding Inverses Algebraically

Guided Example	Steps	List of Inverse Operations
$f(x) = 2x - 3$ $y = 2x - 3$ $x = 2y - 3$ $\frac{x+3}{2} = \frac{2y}{2}$ $f^{-1}(x) = \frac{x+3}{2}$	1 Change f(x) to y. 2 Swap x and y. 3 Solve for y. *(using inverse operations)* 4 Rewrite in function notation. *(if the inverse is a function)* **Remember in modeling/application problems you would not swap the variables, just solve for the other variable.*	$+ \leftrightarrow -$ $\times \leftrightarrow \div$ $(\ )^n \leftrightarrow \sqrt[n]{\ }$ $b^x \leftrightarrow \log_b x$

Example 3: Find the inverse of $h(x) = -3 + x^3$. Determine if the inverse is a function.

① $y = -3 + x^3$

② $x = -3 + y^3$

③ $x = -3 + y^3$

$\quad +3 \quad +3$

$\sqrt[3]{x+3} = \sqrt[3]{y^3}$

$\sqrt[3]{x+3} = y$

④ $\boxed{h^{-1}(x) = \sqrt[3]{x+3}}$

Yes, $h^{-1}(x)$ is a function.

Example 4: Find the inverse of $f(x) = \sqrt{x+7}$ Determine if the inverse is a function.

① $y = \sqrt{x+7}$

② $x = \sqrt{y+7}$

③ $(x)^2 = (\sqrt{y+7})^2$

$x^2 = y + 7$

$-7 \qquad -7$

$x^2 - 7 = y$

④ $\boxed{f^{-1}(x) = x^2 - 7 \text{ for } x \geq 0}$ ← domain restriction

Yes, $f^{-1}(x)$ is a function.

Finding Inverses from Tables

To find the inverse relation from a table, swap the x and y coordinates.

Example 5: Given the partial table below for the function, $f(x)$, find a partial table for the inverse function, $f^{-1}(x)$. Then find $f^{-1}(4)$.

x	0	1	2	3	4
y	6	4	2	0	-2

x	6	4	2	0	-2
y	0	1	2	3	4

← table for $f^{-1}(x)$

$\boxed{f^{-1}(4) = 1}$

Finally, if you want to know if two functions are inverses of each other, you can graph them both and check to see if they are reflections of each other in the line $y = x$, or you can prove they are inverses using **compositions**.

Verifying Inverses Using Compositions

Two functions, $f(x)$ and $g(x)$, are inverses of each other if $f(g(x)) = g(f(x)) = x$. This means that you must be able to compose the functions in both directions and each time, the result is x; conceptually this means that all the operation(s) of one function *undoes* the operation(s) of the other function.

Example 6: Verify the functions $f(x)$ and $g(x)$ are inverses of each other using compositions.

$$f(x) = 2x^3 - 1 \qquad g(x) = \sqrt[3]{\frac{x+1}{2}}$$

$f(g(x))$

$f(g(x))$

$f\left(\sqrt[3]{\frac{x+1}{2}}\right)$

$2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$

$\cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$

$x + 1 - 1$

x

$g(f(x))$

$g(f(x))$

$g(2x^3 - 1)$

$\sqrt[3]{\frac{2x^3 \cancel{-1} \cancel{+1}}{2}}$

$\sqrt[3]{\frac{\cancel{2}x^3}{\cancel{2}}}$

$\sqrt[3]{x^3}$

x

since $f(g(x))$ and $g(f(x)) = x$

f and g are inverses of each other.

Example 7: Explain why $f(x)$ and $g(x)$ are not inverses of each other.

$$f(x) = \frac{x}{2} - 3 \qquad g(x) = 2x + 3$$

$f(g(x))$

$f(2x+3)$

$\frac{2x+3}{2} - 3$

$\frac{2x}{2} + \frac{3}{2} - 3$

$x - \frac{3}{2}$

f & g are not inverses of each other because $f(g(x)) \neq x$.