Name: _____ Date: _____ Period: _____

2.2 Inverses of Functions

Engage: Inverse Relationships

In mathematics, there are basic operations that can be considered inverse operations. Addition and subtraction are inverse operations. The same is true for multiplication and division. Although every function does not necessarily have an inverse that is a function too, we can determine the inverse of a function by simply switching the input and output values. The symbol we use to denote the inverse of $f(x)$ is $f^{-1}(x)$. ← "f inverse of x"

1) The table below represents a function C that computes the percentage of the time that the sky is cloudy in Augusta, Georgia, where x corresponds to the standard numbers for the months.

Cloudy Skies in Augusta												
x(month)	1	2	3	4	5	6	7	8	9	10	11	12
C(x) %	43	40	39	29	28	26	27	25	30	26	31	39

(a) Construct a table of the inverse for this relationship.

%	43	40	39	29	28	26	27	25	30	26	31	39	X
month	1	2	3	4	5	6	7	8	9	10	11	12	Y

b) Determine if the inverse of C(x) is a function. Justify your response.

No, because 39 has two outputs.

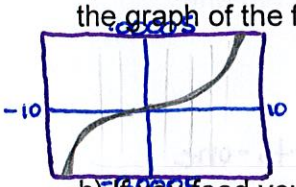
Vocab A function is a relation where each input has exactly one output.

Explore: Weighty Pets

Calories in food provide essential energy, for you as well as your pet. But how much should you be feeding your pet and how does the size and weight of your pet influence the amount of food they need?

1) The adult weight of your dog is determined by the number of calories you feed your puppy. The function $w(k) = \left(\frac{k}{240}\right)^3$ relates the number of kilocalories per day, *k*, to the adult weight of the dog (in kilograms), *w*.

a) Using a graphing calculator, graph w(k). Choose an appropriate window to view the graph of the function. Does the graph of w(k) pass the Vertical Line Test? Explain.



Yes, the graph passes the Vertical Line Test; each vertical line only passes through the graph once.

Xmin = −10
Xmax = 10
Xscl= 1
Ymin= -0.00005
Ymax= 0.00005
Yscl= 0.000005

b) If you feed your puppy 800 kcal a day, what is their predicted adult weight? Round to the nearest tenth.

$W(800) = \left(\frac{800}{240}\right)^3 = 37.0\text{ kg}$

c) Complete the table to predict the adult weight of your pet with each of the different diet plans. Round to the nearest tenth.

(kcal a day)	600	800	1000
(weight in kg)	15.6	37.0	72.3

d) If you want your dog to weigh 15.6 kg, how many calories should you feed him a day?

You should feed him 600 kcal a day.

2) The ideal adult weight for a golden retriever is about 30 kg. How many calories a day should you feed a golden retriever to achieve this ideal weight? Show how you got your answer.

$30 = \left(\frac{k}{240}\right)^3 \Rightarrow \sqrt[3]{30} = \sqrt[3]{\left(\frac{k}{240}\right)^3} \Rightarrow 240\sqrt[3]{30} = \frac{k}{240} \cdot 240$

$k = 240 \cdot \sqrt[3]{30} \approx 746$ kCal

3) The ideal adult weight for a Schnauzer is about 9 kg. How many calories a day should you feed a Schnauzer to achieve this ideal weight? Show how you got your answer.

$9 = \left(\frac{k}{240}\right)^3 \Rightarrow \sqrt[3]{9} = \sqrt[3]{\left(\frac{k}{240}\right)^3} \Rightarrow 240\sqrt[3]{9} = \frac{k}{240} \cdot 240$

$k = 240\sqrt[3]{9} \approx 499$ kCal

4) Write an equation $k(w)$ that outputs the number of calories you should feed a dog whose ideal weight is w kg.

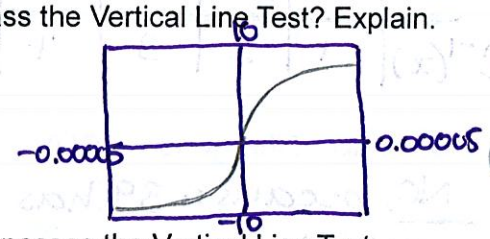
$k(w) = 240\sqrt[3]{w}$

$\sqrt[3]{w} = \sqrt[3]{\left(\frac{k}{240}\right)^3}$

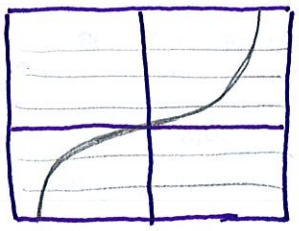
$240\sqrt[3]{w} = \frac{k}{240} \cdot 240$

5) Using a graphing calculator, graph k(w). Does the graph of k(w) pass the Vertical Line Test? Explain.

Yes, the graph passes the VLT; every line only touches the graph once.



6) Revisit the graph of w(k). In #1 part a, you noted the graph of w(k) passes the Vertical Line Test. Sketch the graph of w(k) in the space below. Draw Horizontal Lines across the graph. At most, how many times does each of the horizontal lines you have drawn intersect the graph of w(k)? Explain.



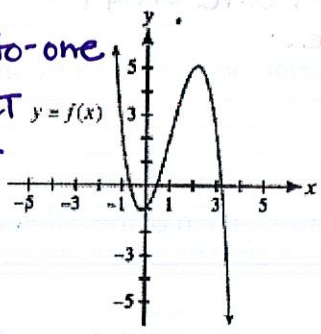
Each horizontal line touches/intersects the graph only once. So, W(k) also passes the Horizontal Line Test.

A graphical way to determine if a function is one-to-one (the inverse is a function) is the Horizontal Line Test. If every horizontal line intersects the graph of a function f at most once, then f is a one-to-one function. In other words, the inverse is also a function.

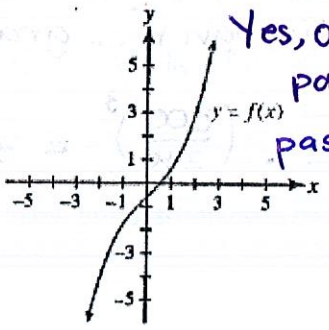
7) Use each graph to determine if f is one-to-one and if f has an inverse function.

(a)

Not one-to-one
passes VLT
fails HLT



(b)



Yes, one-to-one
passes VLT
passes HLT