

2.1 Compositions of Functions (continued)

Composition of Functions is just another operation like adding, subtracting, multiplying and dividing.

Please note the notations for adding, subtracting, multiplying and dividing functions:

Operation	Add	Subtract	Multiply	Divide
What to Do	Combine the like terms of the expressions together.	Distribute the minus to every term in the second function; then combine like terms.	Distribute or FOIL based on the number of terms multiplying. Simplify.	Write a fraction with the two expressions.
Notations	$f + g$	$f - g$	gf	$\frac{g}{f}$
	$(f + g)(x)$	$(f - g)(x)$	$(g \cdot f)(x)$	$\left(\frac{g}{f}\right)(x)$ or $(g \div f)(x)$
	$f(x) + g(x)$	$f(x) - g(x)$	$g(x) \cdot f(x)$	$\frac{g(x)}{f(x)}$

say "f of g of x"

The notation for compositions are $f(g(x))$ or $(f \circ g)(x)$. Notice how the circle is open between the two function letters.

1) Suppose $f(x) = 3x - 5$, $g(x) = x^2$, $h(x) = 4x^3 + 7$

a) Find $f(g(2))$

$g(2) = 2^2$
 $g(2) = 4$
 $f(4) = 3(4) - 5$
 $f(4) = 7$

7

b) Find $(g \circ h)(1)$

$h(1) = 4(1)^3 + 7$
 $h(1) = 11$
 $g(11) = 11^2$
 $g(11) = 121$

121

2) Suppose you are given the table below.

a) Find $f(g(1)) =$ 3

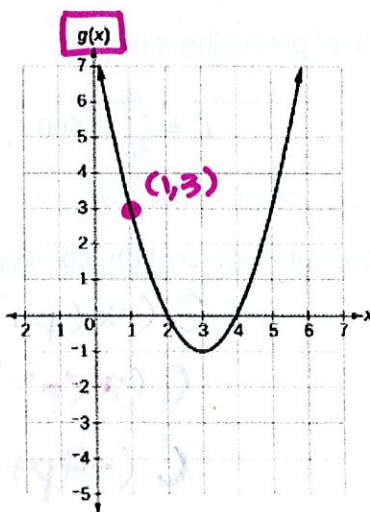
$g(1) = 3$
 $f(3) = 3$

x	f(x)	g(x)
1	6	3
2	8	5
3	3	2
4	1	7

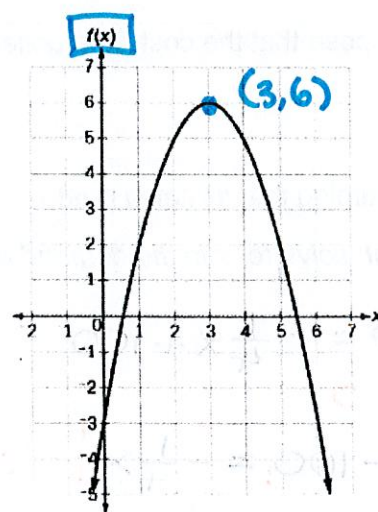
b) Find $(g \circ f)(3) =$ 2

$f(3) = 3$
 $g(3) = 2$

3) Suppose you are given the graphs on the right. Find $f(g(1))$.



(a)



(b)

$g(1) = 3$
 $f(3) = 6$

$f(g(1)) =$ 6

INSIDE FUNCTION

4) OUTSIDE function

Suppose $f(x) = 2x + 3$ and $g(x) = -4x^2 + 12$ and $h(x) = x^2 + 5x$. Find the compositions.

$$\begin{aligned} \text{a) } f(g(x)) &= f(g(x)) \\ &= f(-4x^2 + 12) \\ &= 2(-4x^2 + 12) + 3 \\ &= -8x^2 + 24 + 3 \end{aligned}$$

$$f(g(x)) = -8x^2 + 27$$

$$\begin{aligned} \text{c) } (h \circ f)(x) &= h(f(x)) \\ &= h(2x + 3) \\ &= (2x + 3)^2 + 5(2x + 3) \\ &= 4x^2 + 12x + 9 + 10x + 15 \end{aligned}$$

$$(h \circ f)(x) = 4x^2 + 22x + 24$$

$$\begin{aligned} \text{b) } g(f(x)) &= g(f(x)) \\ &= g(2x + 3) \\ &= -4(2x + 3)^2 + 12 \\ &= -4(4x^2 + 12x + 9) + 12 \\ &= -16x^2 - 48x - 36 + 12 \end{aligned}$$

$$g(f(x)) = -16x^2 - 48x - 24$$

NOTE: $(2x+3)^2$
 $(2x+3)(2x+3)$ FOIL
 $4x^2 + 6x + 6x + 9$
 $4x^2 + 12x + 9$

5) The price p , in dollars of a certain product and the quantity x sold follow the demand equation

$$p = -\frac{1}{4}x + 100 \quad 0 \leq x \leq 400$$

Suppose that the cost C , in dollars, of producing x units is

$$C = \frac{\sqrt{x}}{25} + 600$$

Assuming that all items produced are sold, find the cost C as a function of the price p . $C(p)$

[Hint: solve for x in the demand equation then form the composite function]

$$\begin{aligned} p &= -\frac{1}{4}x + 100 \\ -100 & \quad -100 \\ -4[p - 100] &= \left[-\frac{1}{4}x\right] - 4 \end{aligned}$$

$$-4p + 400 = x$$

$$\uparrow$$

$$x(p)$$

$$C(x(p))$$

$$C(x(p))$$

$$C(-4p + 400)$$

$$\frac{\sqrt{-4p + 400}}{25} + 600$$

* There is nothing to simplify so leave it

$$C(p) = \frac{\sqrt{-4p + 400}}{25} + 600$$

CALCULATOR STEPS

Press $\boxed{Y=}$

Use
 $\boxed{2nd}$
 \boxed{mode}

$Y_1 = 3x - 5$
 $Y_2 = x^2$
 $Y_3 = 4x^3 + 7$ } enter $f(x), g(x), h(x)$
into Y_1, Y_2, Y_3
respectfully

to get back
to home
screen.

Shortcuts for functions

$\boxed{\alpha}$ \boxed{trace}

On the home screen...

$Y_1(Y_2(2))$ is the same as $f(g(2))$

$Y_2(Y_3(1))$ is the same as $(g \circ h)(1)$

Use the calculator steps to check your work
on the evaluate problems of compositions of
functions.