

Example 1 Square Roots of Negative Numbers

Simplify.

a. $\sqrt{-72}$

$$\begin{aligned}\sqrt{-72} &= \sqrt{-1 \cdot 2 \cdot 6^2} \\ &= \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{6^2} \\ &= i \cdot 6 \cdot \sqrt{2} = 6i\sqrt{2}\end{aligned}$$

b. $\sqrt{-108b^7}$

$$\begin{aligned}\sqrt{-108b^7} &= \sqrt{-1 \cdot 6^2 \cdot b^6 \cdot 3b} \\ &= \sqrt{-1} \cdot \sqrt{6^2} \cdot \sqrt{b^6} \cdot \sqrt{3b} \\ &= i \cdot 6 \cdot b^3 \cdot \sqrt{3b} \text{ or } 6b^3i\sqrt{3b}\end{aligned}$$

Example 2 Products of Pure Imaginary Numbers

Simplify.

a. $(-9i) \cdot (-5i)$

$$\begin{aligned}(-9i) \cdot (-5i) &= 45i^2 \\ &= 45(-1) \\ &= -45\end{aligned}$$

Multiply.
 $i^2 = -1$
Simplify.

b. $2\sqrt{-72} \cdot (-3)\sqrt{-50}$

$$\begin{aligned}2\sqrt{-72} \cdot (-3)\sqrt{-50} &= 2i\sqrt{72} \cdot (-3i)\sqrt{50} \\ &= -6i^2\sqrt{3600} \\ &= -6i^2\sqrt{60^2} \\ &= -6(-1)(60) \\ &= 360\end{aligned}$$

$i = \sqrt{-1}$
Multiply.
Simplify.
Multiply.
Simplify.

Example 3 Equation with Pure Imaginary SolutionsSolve $4x^2 + 100 = 0$.

$4x^2 + 100 = 0$

Original equation

$4x^2 = -100$

Subtract 100 from each side.

$x^2 = -25$

Divide each side by 4.

$x = \pm\sqrt{-25}$

Square Root Property

$x = \pm 5i$

$\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1}$

Example 4 Equate Complex NumbersFind the values of a and b that make the equation $a + 4 + (2b - 6)i = 7 + 9i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$a + 4 = 7$

Real parts

$2b - 6 = 9$

Imaginary parts

$a = 3$

Subtract 4 from each side.

$2b = 15$

Add 6 to each side.

$b = 7.5$

Divide each side by 2.

Example 5 Add and Subtract Complex Numbers

Simplify.

a. $(-7 + 5i) + (12 + 3i)$

$(-7 + 5i) + (12 + 3i)$

$= (-7 + 12) + (5 + 3)i$

$= 5 + 8i$

Commutative and Associative Properties
Simplify.

b. $(6 + 3i) - (-1 - 4i)$

$(6 + 3i) - (-1 - 4i)$

$= [6 - (-1)] + [3 - (-4)]i$

$= 7 + 7i$

Commutative and Associative Properties
Simplify.**Example 6 Multiply Complex Numbers****ELECTRICITY** In an AC circuit, the voltage E , current I , and impedance Z are related by the formula $E = I \cdot Z$. Find the voltage in a circuit with current $2 + 5j$ amps and impedance $8 - 3j$ ohms.

$E = I \cdot Z$

$= (8 - 3j)(2 + 5j)$

$= 8(2) + 8(5j) + (-3j)(2) + (-3j)(5j)$

$= 16 + 40j - 6j - 15j^2$

$= 16 + 34j - 21(-1)$

$= 37 + 34j$

Electricity formula

$I = 8 - 3j, Z = 2 + 5j$

FOIL

Multiply.

$j^2 = -1$

Add.

The voltage is $37 + 34j$ volts.**Example 7 Divide Complex Numbers****Simplify.**

a. $\frac{-2}{3 + 5i}$

b. $\frac{6 - 7i}{3i}$

$\frac{-2}{3 + 5i} = \frac{-2}{3 + 5i} \cdot \frac{3 - 5i}{3 - 5i}$

 $3 + 5i$ and $3 - 5i$
are conjugates.

$\frac{6 - 7i}{3i} = \frac{6 - 7i}{3i} \cdot \frac{-3i}{-3i}$ Multiply by $\frac{-3i}{-3i}$.

$= \frac{-6 + 10i}{9 - 25i^2}$

Multiply.

$= \frac{-18i + 21i^2}{-9i^2}$ Multiply.

$= \frac{-6 + 10i}{34}$

$i^2 = -1$

$= \frac{-21 - 18i}{9}$ $i^2 = -1$

$= -\frac{3}{17} + \frac{5}{17}i$

Standard form

$= \frac{-7}{3} - 2i$ Standard form