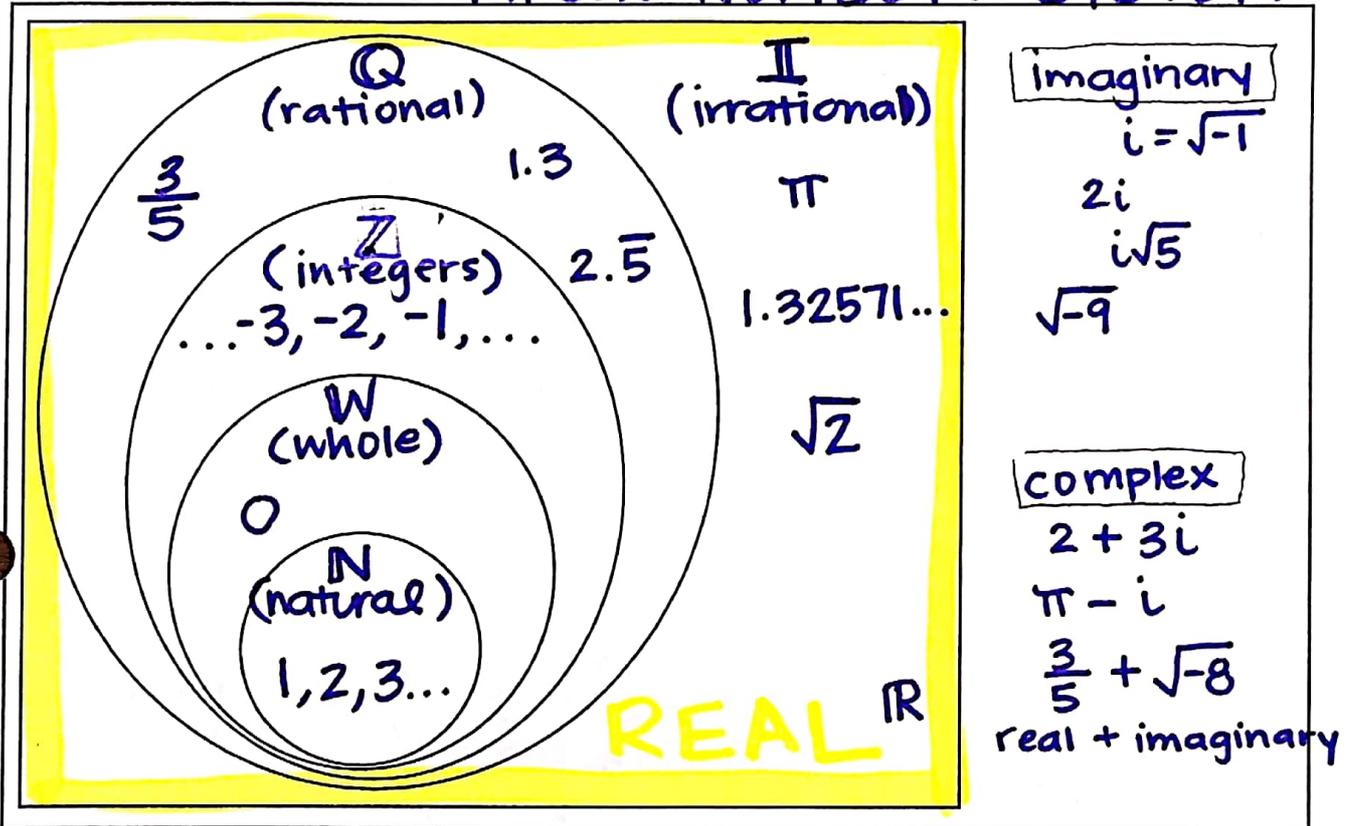


1.5 Real and Imaginary Numbers Notes

You have been told in the past that you are not allowed to have a negative number under a radical.

This isn't always true. There is another system of numbers that are called the imaginary numbers. This system includes all radicals with a negative number in it.

COMPLEX NUMBER SYSTEM



This system uses the lowercase letter i to represent the negative under a radical.

$i = \sqrt{-1}$ Note: $i^2 = -1$

These radicals are rewritten by ~~making the number positive~~, simplifying the radical if possible, and putting the i in front of the radical. The i goes after the number if it is a perfect square.

Ex.

1. $\sqrt{-4} =$

$\sqrt{-1} \cdot \sqrt{4}$

$i \cdot 2$

$2i$

2. $\sqrt{-36} =$

$\sqrt{-1} \cdot \sqrt{36}$

$i \cdot 6$

$6i$

3. $\sqrt{-3} =$

$\sqrt{-1} \cdot \sqrt{3}$

$i\sqrt{3}$

4. $\sqrt{-11} =$

$\sqrt{-1} \cdot \sqrt{11}$

$i\sqrt{11}$

Definitions

1. Imaginary Number: $i^2 = -1$; If a is a positive real number, then the principal square root of negative a is the imaginary number $i\sqrt{a}$; that is, $\sqrt{-a} = i\sqrt{a}$.
(Extends the existing number system by giving a solution to problems that look like: $x^2 = -1$.)
2. Complex Number – is a number in the form $a+bi$, where a and b are real numbers and $i = \sqrt{-1}$. The number a is the real part of $a+bi$ and bi is the imaginary part. Any solution should be written in this form. (It is customary to put i in front of a radical if it is part of the solution.)
3. Patterns of i - To determine the value of i to any power just find the largest multiple of 4 and i to that multiple of 4 = 1 then the remainder is the exponent of i that is evaluated and multiplied by 1. Example: $i^{347} = i^{344}i^3 = (1)(-i) = -i$.

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

Guided Practice Part I: Complete all problems and show ALL work. Box answers.

1. $\sqrt{-16}$

$$\boxed{4i}$$

2. $\sqrt{-28}$

$$i\sqrt{28}$$

$$\boxed{2i\sqrt{7}}$$

$$\begin{array}{r} 28 \\ \wedge \\ 4 \quad 7 \\ \wedge \\ \underline{2} \quad \underline{2} \end{array}$$

3. $\sqrt{-5}$

$$\boxed{i\sqrt{5}}$$

4. $\sqrt{-10} \cdot \sqrt{-2}$

$$\sqrt{20}$$

$$\boxed{2\sqrt{5}}$$

$$\begin{array}{r} 20 \\ \wedge \\ 10 \quad 2 \\ \wedge \\ \underline{5} \quad \underline{2} \end{array}$$

5. $\frac{\sqrt{-50}}{\sqrt{-10}}$

$$= \sqrt{\frac{-50}{-10}}$$

$$= \boxed{\sqrt{5}}$$

6. $\frac{3\sqrt{21}\sqrt{-6}}{1\sqrt{2}} = 3\sqrt{\frac{-6}{2}} = 3\sqrt{-3} = \boxed{3i\sqrt{3}}$

7. $(3\sqrt{-5})^2 = 3\sqrt{-5} \cdot 3\sqrt{-5} = 9\sqrt{25} = 9 \cdot 5 = \boxed{45}$

8. $(6\sqrt{-8})^2 = 6\sqrt{-8} \cdot 6\sqrt{-8} = 36\sqrt{64} = 36 \cdot 8 = \boxed{288}$

9. $\sqrt[3]{-27} = \sqrt[3]{-1} \cdot \sqrt[3]{27} = -1 \cdot 3 = -3$

$\sqrt[3]{-1} = -1$
because $(-1)^3 = -1$
 $\sqrt[3]{27} = 3$
because $3^3 = 27$